Modelling of turbulent natural convection

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Modelling of wall bounded turbulent natural convection
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Outline of the presentation

✓ **Introduction**

- ▷ Influence of buoyancy in the equations of motion
- ▷ Convection regimes

✓ **Influence of buoyancy on turbulence: physics → modelling**

- ▷ Dynamics: energy, anisotropy, redistribution, dissipation
- ▷ Heat fluxes

✓ **The devil is in the detail**

- ▷ Dissipation
- ▷ Time scales
- ▷ Transition/Relaminarization

✓ **Simplified models?**

- ▷ Algebraic flux models
- ▷ Eddy-viscosity models
- ▷ Variable Pr_t?

✓ **Unsteady approaches**

✓ **Conclusion**

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Equations of motion

φ^* denotes an instantaneous variable

✓ Mass conservation (continuity)

$$\frac{\partial \rho^*}{\partial t} + \frac{\partial \rho^* u_i^*}{\partial x_i} = 0$$

✓ Momentum conservation

$$\frac{\partial \rho^* u_i^*}{\partial t} + \frac{\partial \rho^* u_i^* u_j^*}{\partial x_j} = -\frac{\partial p^*}{\partial x_i} + \frac{\partial}{\partial x_j}\left[\mu\left(\frac{\partial u_i^*}{\partial x_j} + \frac{\partial u_j^*}{\partial x_i}\right)\right] + \rho^* g_i$$

✓ Energy conservation

$$\rho^* C_v \frac{\partial T^*}{\partial t} + \rho^* C_v \frac{\partial u_i^* T^*}{\partial x_i} = \frac{\partial}{\partial x_i}\left(k \frac{\partial T^*}{\partial x_i}\right) + 2\mu s_{ij}^* s_{ij}^*$$

Temperature variations directly affect the terms in red.

Boussinesq approximation

✓ Assuming that the temperature differences are not too large, the **Boussinesq approximation** can be applied:

- ▷ Density variations can be neglected ($\rho^* = \rho_0$)
- ▷ Except in the buoyancy term $\rho^* g_i$.

✓ Additional simplification: in this case, it is standard to also assume that μ, k and C_v are independent of the temperature

$$\frac{\partial u_i^*}{\partial x_i} = 0$$

$$\rho_o \frac{\partial u_i^*}{\partial t} + \rho_0 \frac{\partial u_i^* u_j^*}{\partial x_j} = -\frac{\partial p^*}{\partial x_i} + \mu \frac{\partial^2 u_i^*}{\partial x_j \partial x_j} + \rho^* g_i$$

$$\rho_0 C_v \frac{\partial T^*}{\partial t} + \rho_0 C_v \frac{\partial u_i^* T^*}{\partial x_i} = k \frac{\partial^2 T^*}{\partial x_i \partial x_i}$$

✓ Remark: for low Mach number flows, the heat source due to viscous dissipation $2\mu s_{ij}^* s_{ij}^*$ is neglected.

Linear Boussinesq approximation

✓ Density variations in the buoyancy term are linear in the temperature:

$$\rho^* = \rho_0^* - \beta\rho_0^*(T^* - T_0^*) \quad \text{with } \beta = -\frac{1}{\rho_0^*}\frac{\partial \rho^*}{\partial T}$$

$$\frac{\partial u_i^*}{\partial x_i} = 0$$

$$\frac{\partial u_i^*}{\partial t} + \frac{\partial u_i^* u_j^*}{\partial x_j} = -\frac{1}{\rho_0}\frac{\partial \mathrm{p}^*}{\partial x_i} + \nu\frac{\partial^2 u_i^*}{\partial x_j \partial x_j} - \beta g_i\left(T^* - T_0\right)$$

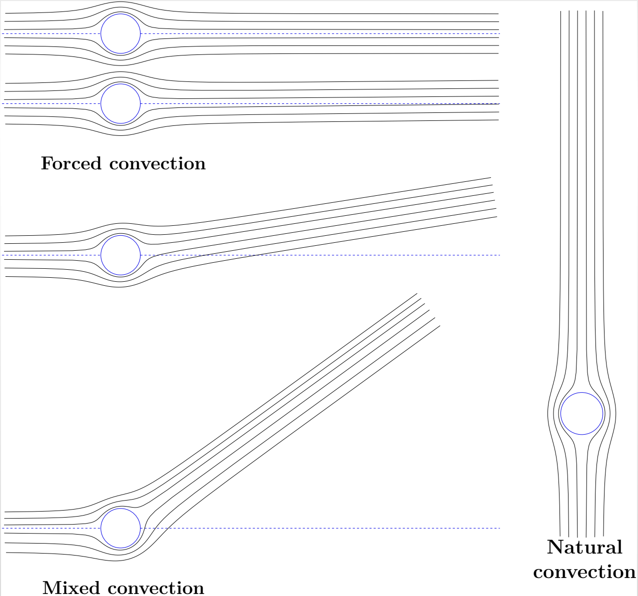
$$\frac{\partial T^*}{\partial t} + \frac{\partial u_i^* T^*}{\partial x_i} = \alpha\frac{\partial^2 T^*}{\partial x_i \partial x_i}$$

✓ Remark: the hydrostatic pressure is contained in p^* :

$$\mathrm{p}^* = p^* - \rho_0 g_j x_i \delta_{ij}$$

The different flow regimes

Forced, mixed, natural convection



Richardson number

- ✓ Relative weight of the buoyant term compared to the convective term: Richardson number

$$\frac{\partial u_i^*}{\partial t} + \frac{\partial u_i^* u_j^*}{\partial x_j} = -\frac{1}{\rho_0}\frac{\partial \mathrm{p}^*}{\partial x_i} + \nu \frac{\partial^2 u_i^*}{\partial x_j \partial x_j} - \beta g_i \left(T^* - T_0\right) \tag{1}$$

$$\frac{\partial u_i^* u_j^*}{\partial x_j} \simeq \frac{U_{\mathrm{ref}}^2}{L_{\mathrm{ref}}} \quad ; \quad \beta g_i \left(T^* - T_0\right) \simeq \beta g \Delta T \Rightarrow \boxed{Ri = \frac{\beta g\, \Delta T\, L_{\mathrm{ref}}}{U_{\mathrm{ref}}^2}}$$

- ✓ The time scale of convective phenomena: $\frac{\partial u_i^*}{\partial t} \simeq \frac{\partial u_i^* u_j^*}{\partial x_j} \Rightarrow \tau_{\mathrm{conv}} = \frac{L_{\mathrm{ref}}}{U_{\mathrm{ref}}}$

- ✓ Time scale of buoyancy phenomena: $\frac{\partial u_i^*}{\partial t} \simeq \beta g_i \left(T^* - T_0\right) \Rightarrow \tau_{\mathrm{buo}} = \frac{U_{\mathrm{ref}}}{\beta g \Delta T}$

$$\Rightarrow Ri = \frac{\tau_{\mathrm{conv}}}{\tau_{\mathrm{buo}}}$$

- ✓ When the Richardson number is small compared to unity ($Ri << 1$), it is considered that the flow is in the forced convection regime.

- ✓ In this case, heat transfer has no influence on dynamics, the model for the Reynolds stress does not require modifications.

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Reylnolds decomposition

✓ Reynolds decomposition: $u_i^* = U_i + u_i$; $\mathrm{p}^* = P + p$; $T^* = T + \theta$

$$\frac{\partial U_i}{\partial x_i} = 0$$

$$\frac{\partial U_i}{\partial t} + U_j \frac{\partial U_i}{\partial x_j} = -\frac{1}{\rho}\frac{\partial P}{\partial x_i} + \nu \frac{\partial^2 U_i}{\partial x_j \partial x_j} - \frac{\partial \overline{u_i u_j}}{\partial x_j} - \beta g_i \left(T - T_0\right)$$

$$\frac{\partial T}{\partial t} + U_i \frac{\partial T}{\partial x_i} = \alpha \frac{\partial^2 T}{\partial x_i \partial x_i} - \frac{\partial \overline{u_i \theta}}{\partial x_i}$$

Influence of buoyancy on turbulence : dynamics

Buoyancy production

Transport equation for the Reynolds stress tensor:

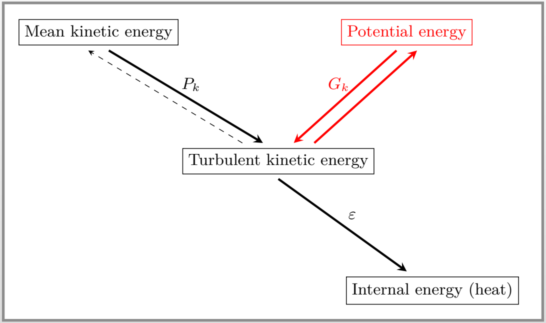
$$\frac{\mathrm{D}\overline{u_i u_j}}{\mathrm{D}t} = P_{ij} + G_{ij} + \phi_{ij} - \varepsilon_{ij} + D_{ij}^{\nu,t,p}$$

✓ Buoyancy production: $G_{ij} = -\beta g_i \overline{u_j \theta} - \beta g_j \overline{u_i \theta}$

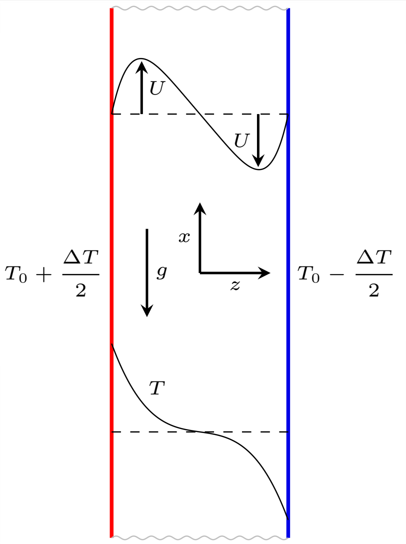
Transport equation for k

$$\frac{\mathrm{D}k}{\mathrm{D}t} = P_k + G_k - \varepsilon + D^{\nu,t,p}$$

- ✓ Buoyancy production: $G_k = \frac{1}{2} G_{ii} = -\beta g_i \overline{u_i \theta}$
- ✓ Energy cascade:



Example: differentially heated vertical channel



= natural convection

Momentum: $$0 = \nu \frac{\partial^2 U}{\partial z^2} - \frac{\partial \overline{uw}}{\partial z} + \beta g (T - T_0)$$

Temperature: $$0 = \frac{\nu}{Pr} \frac{\partial^2 T}{\partial z^2} - \frac{\partial \overline{w\theta}}{\partial z}$$

- ✓ Boundary-layer-like: $\overline{uw}$ and $\overline{w\theta}$ only active components.
  - ▷ $\overline{w\theta}$ determines the temperature profile ($\overline{w\theta} = 0 \Rightarrow$ laminar profile)
  - ▷ $\overline{uw}$ and T determine the velocity profile

✓ Production:

Component	Strain production	Buoyancy production
$\overline{u^2}$	$-2\overline{uw}\dfrac{\partial U}{\partial z}$	$+2\beta g\overline{u\theta}$
$\overline{v^2}$	0	0
$\overline{w^2}$	0	0
$\overline{uw}$	$-\overline{w^2}\dfrac{\partial U}{\partial z}$	$+\beta g\overline{w\theta}$
k	$-\overline{uw}\dfrac{\partial U}{\partial z}$	$+\beta g\overline{u\theta}$

✓ Buoyancy modifies the turbulent energy (potential energy $\rightleftarrows$ turbulent energy)

✓ The vertical direction is a particular direction $\Rightarrow$ buoyancy generates anisotropy

✓ Dynamics and thermal turbulence are strongly coupled

Redistribution

✓ Chou's analysis is modified

$$\phi_{ij} = \underbrace{\phi_{ij}^1}_{\text{Slow term}} + \underbrace{\phi_{ij}^2}_{\text{Rapid term}} + \underbrace{\phi_{ij}^3}_{\text{Buoyant term}}$$

▷ Rapid term

Production: $P_{ij} = -\overline{u_i u_k}\dfrac{\partial U_j}{\partial x_k}$ IP model: $\phi_{ij}^2 = -C_2\left(P_{ij} - \dfrac{2}{3}P\delta_{ij}\right)$

▷ Buoyant term: similar

Production: $G_{ij} = -\beta g_i \overline{u_j\theta} - \beta g_j \overline{u_i\theta}$ IP model: $\phi_{ij}^3 = -C_3\left(G_{ij} - \dfrac{2}{3}G\delta_{ij}\right)$

Dissipation

✓ Buoyancy appears in the exact equations for ε_{ij} and ε.

✓ For ε: $$G_\varepsilon = -2\beta g_i \nu \overline{\frac{\partial u_i}{\partial x_k}\frac{\partial \theta}{\partial x_k}}$$

✓ No consensus in the literature: see next part

Influence of buoyancy on turbulence : heat fluxes

Transport equation for the turbulent heat flux:

$$\frac{\partial \overline{u_i\theta}}{\partial t} + U_k \frac{\partial \overline{u_i\theta}}{\partial x_k} = P_{i\theta}^U + P_{i\theta}^T + G_{i\theta} + \phi_{i\theta} - \varepsilon_{i\theta} + D_{i\theta}^{\nu,t,p}$$

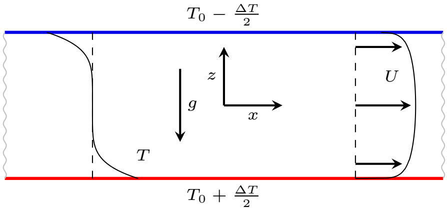
✓ Three different production mechanisms:

- ▷ By strain: $P_{i\theta}^U = -\overline{u_k\theta}\frac{\partial U_i}{\partial x_k}$
- ▷ By temperature gradient: $P_{i\theta}^T = -\overline{u_i u_k}\frac{\partial T}{\partial x_k}$
- ▷ By buoyancy: $G_{i\theta} = -\beta g_i \overline{\theta^2}$

✓ Production strongly couples velocity and temperature fields

✓ New variable to solve: the temperature variance $\overline{\theta^2}$

Example: Unstably stratified channel flow

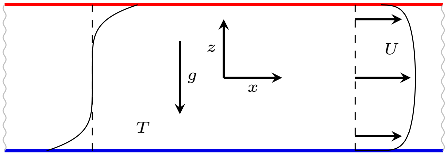


Comp.	Strain prod.	Buo. prod.
$\overline{u^2}$	$-2\overline{uw}\frac{\partial U}{\partial z}$	0
$\overline{v^2}$	0	0
$\overline{w^2}$	0	$2\beta g\overline{w\theta}$
k	$-\overline{uw}\frac{\partial U}{\partial z}$	$\beta g\overline{w\theta}$

Comp.	Strain prod.	T gradient prod.	Buo. prod.
$\overline{u\theta}$	$-\overline{w\theta}\frac{\partial U}{\partial z}$	$-\overline{uw}\frac{\partial T}{\partial z}$	0
$\overline{v\theta}$	0	0	0
$\overline{w\theta}$	0	$-\overline{w^2}\frac{\partial T}{\partial z}$	$\beta g\overline{\theta^2}$

$\Rightarrow$ Unstable stratification promotes vertical fluctuations.

Example: Stably stratified channel flow



Comp.	Strain prod.	Buo. prod.
$\overline{u^2}$	$-2\overline{uw}\dfrac{\partial U}{\partial z}$	0
$\overline{v^2}$	0	0
$\overline{w^2}$	0	$2\beta g\overline{w\theta}$
k	$-\overline{uw}\dfrac{\partial U}{\partial z}$	$\beta g\overline{w\theta}$

Comp.	Strain prod.	T gradient prod.	Buo. prod.
$\overline{u\theta}$	$-\overline{w\theta}\dfrac{\partial U}{\partial z}$	$-\overline{uw}\dfrac{\partial T}{\partial z}$	0
$\overline{v\theta}$	0	0	0
$\overline{w\theta}$	0	$-\overline{w^2}\dfrac{\partial T}{\partial z}$	$\beta g\overline{\theta^2}$

⇒ Stable stratification damps vertical fluctuations.

✓ Can lead to 2C turbulence in the atmosphere or the ocean.

✓ Can lead to relaminarization

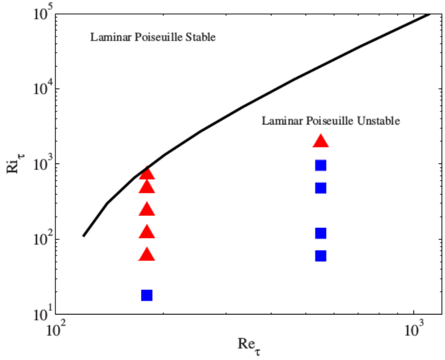


FIG. 3. (Color online) Phase diagram showing the regime in stably stratified channel flow as a function of friction Reynolds number and Richardson number Re_τ and Ri_τ. Squares: the walls remain turbulent. Triangles: laminar and turbulent patches coexist in the near-wall region. The solid lines are the neutral curve obtained from linear stability analysis (Ref. 34).

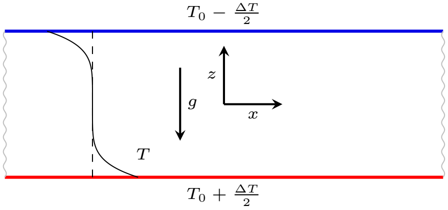
DNS of stably stratified channel flows

From García-Villalba & del Álamo (2011)

$$Re_\tau = \frac{h u_\tau}{\nu}$$

$$Ri_\tau = \frac{\beta g \, \Delta T \, h}{u_\tau^2}$$

Example: Rayleigh-Bénard flow (natural convection)



Comp.	Strain prod.	Buo. prod.
$\overline{u^2}$	0	0
$\overline{v^2}$	0	0
$\overline{w^2}$	0	$2\beta g\overline{w\theta}$
k	0	$\beta g\overline{w\theta}$

Comp.	Strain prod.	T gradient prod.	Buo. prod.
$\overline{u\theta}$	0	0	0
$\overline{v\theta}$	0	0	0
$\overline{w\theta}$	0	$-\overline{w^2}\frac{\partial T}{\partial z}$	$\beta g\overline{\theta^2}$

✓ Turbulence is produced by the buoyancy term $\beta g\overline{\theta^2}$ only.

✓ If $\beta g\overline{\theta^2}$ is neglected (SGDH or GGDH) $\Rightarrow$ laminar (linear) temperature profile.

Influence of buoyancy on the other terms

$$\frac{\partial \overline{u_i\theta}}{\partial t} + U_k \frac{\partial \overline{u_i\theta}}{\partial x_k} = P_{i\theta}^U + P_{i\theta}^T + G_{i\theta} + \phi_{i\theta} - \varepsilon_{i\theta} + D_{i\theta}^{\nu,t,p}$$

- ✓ Scrambling term $\phi_{i\theta}$
  - ▷ Chou's analysis is modified by buoyancy:

$$\phi_{i\theta} = \phi_{i\theta}^1 + \phi_{i\theta}^2 + \phi_{i\theta}^3$$

  - ▷ As usual: isotropization of production

$$\phi_{i\theta}^3 = -C_{\theta 3} G_{i\theta} = C_{\theta 3} \beta g_i \overline{\theta^2}$$

- ✓ Many different models in the literature
  - ▷ Hanjalić, K. (2002) One-point closure models for buoyancy-driven turbulent flows. *Annu. Rev. Fluid Mech.* **34**, 321–347
  - ▷ Hanjalić, K. & Launder, B. (2011) *Modelling Turbulence in Engineering and the Environment. Second-Moment Routes to Closure*. Cambridge University Press

Influence of buoyancy on turbulence

Consequences for modelling

✓ Buoyancy strongly couples fluctuating velocity and temperature fields.

✓ The vertical direction is obviously a privileged direction.

✓ **Potential energy** $\leftrightarrows$ **turbulent energy** transfer depends on stratification (stable/unstable/neutral).

✓ Buoyancy production generates anisotropic turbulence (Reynolds stress and turbulent heat flux): damps or promotes vertical fluctuations.

✓ Buoyancy also affects redistribution, dissipation, scrambling.

⇒ Second moment closure is the natural level to account for these phenomena: Reynolds stress model (RSM) + Differential flux model (DSM).

✓ Are simplified models relevant:

- ▷ RSM+Algebraic Flux Model (AFM)?
- ▷ RSM+gradient models (SGDH, GGDH)?
- ▷ Eddy-viscosity models (EVM)?

→ ERCOFTAC SIG-15 Workshop!

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Influence of buoyancy on dissipation

✓ ε-equation: $$\frac{\mathrm{d}\varepsilon}{\mathrm{d}t} = C_{\varepsilon 1}\frac{\varepsilon}{k}P_k + G_\varepsilon - C_{\varepsilon 2}\frac{\varepsilon^2}{k} + D_\varepsilon$$

✓ No consensus in the literature: does buoyancy have on influence on the energy cascade $\Rightarrow$ on the dissipation. How?

▷ No influence: $G_\varepsilon = 0$

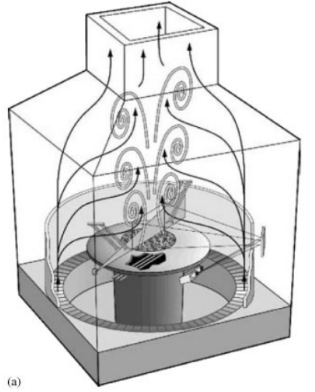
▷ Same influence as P_k: $G_\varepsilon = C_{\varepsilon 3}\frac{\varepsilon}{k}G_k$
with $C_{\varepsilon 3} = C_{\varepsilon 1}$ or $C_{\varepsilon 3} < C_{\varepsilon 1}$

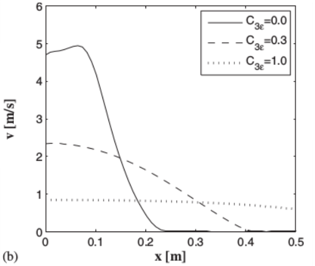
▷ Unstable: influence; Stable: no influence: $G_\varepsilon = C_{\varepsilon 3}\frac{\varepsilon}{k}\max(G_k; 0)$

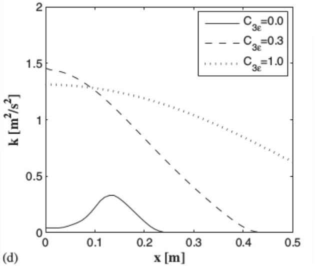
▷ Influence function of the flux Richardson number:
$C_{\varepsilon 1}\frac{\varepsilon}{k}(P_k + G_k)(1 + C_{\varepsilon 3}Ri_f)$
with $Ri_f = -\frac{G_k}{P_k + G_k}$

Example: buoyant plume of Chung & Devaud (2008)

$$G_\varepsilon = C_{\varepsilon 1}(1 - C_{\varepsilon 3})\frac{\varepsilon}{k}G_k$$







Time scales

$$\frac{\mathrm{d}\varepsilon}{\mathrm{d}t} = C_{\varepsilon 1}\frac{P_k}{\tau} + C_{\varepsilon 3}\frac{G_k}{\tau'} - C_{\varepsilon 2}\frac{\varepsilon^2}{k} + D_\varepsilon$$

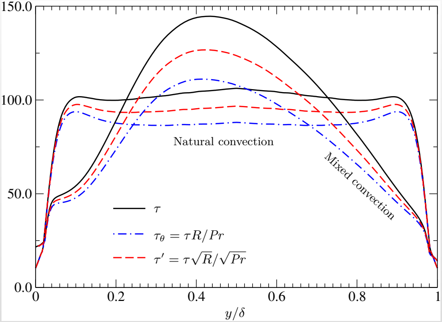
Same time scale?

✓ G_ε is exactly $6\nu \frac{G_k}{\lambda_{u\theta}^2}$ $\lambda_{u\theta}^2$ = Taylor micro-scale associated to $\overline{u_i(\mathbf{x})\theta(\mathbf{x}+\mathbf{r})}$

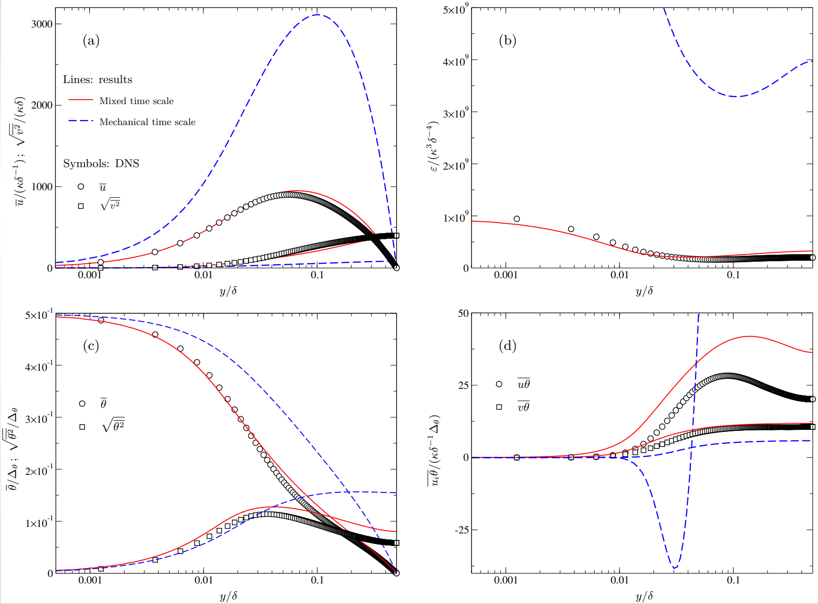
⇒ It can be shown that $\tau' = C_{\tau 1}\tau + C_{\tau 2}\frac{\tau_\theta}{Pr}$ $\tau = \frac{k}{\varepsilon}$; $\tau_\theta = \frac{\overline{\theta^2}}{\varepsilon_{\theta^2}}$

✓ Other mixed time scales: $\tau' = \sqrt{\tau^2 + \tau_\theta^2}$

or $\tau' = \frac{\sqrt{\tau\tau_\theta}}{\sqrt{Pr}}$
(Dehoux *et al.*, 2017)



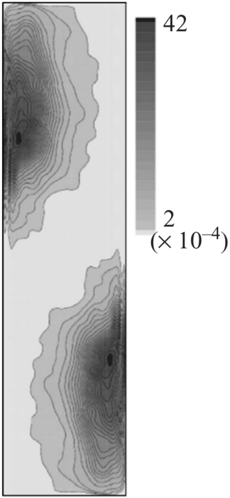
A priori tests



EB-RSM+EB-DFM

From Dehoux *et al.* (2017)

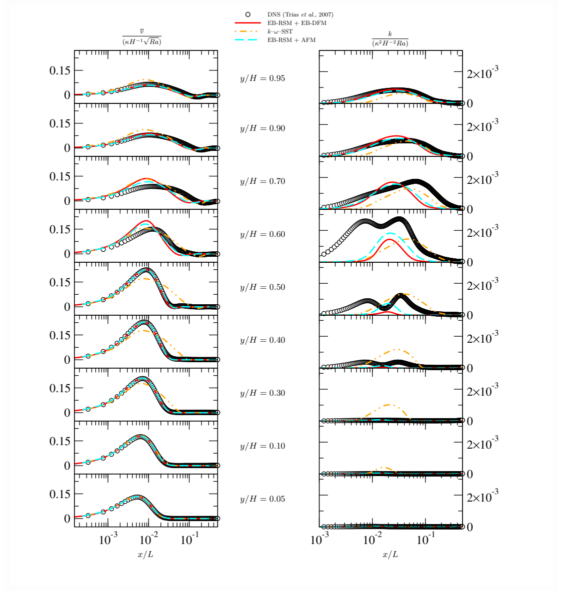
Transition/relaminarization



Turbulent kinetic energy

DNS of a differentially heated cavity at $Ra = 10^{10}$
From Trias *et al.* (2007)

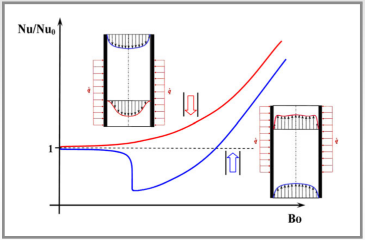
- ✓ Buoyancy effects can lead to co-existing laminar and turbulent regions
- ✓ RANS model are not designed to represent such phenomena
- ✓ The location of transition/relaminarization depends on (uncontrolled) modelling subtleties



Trias *et al.* (2007)'s cavity

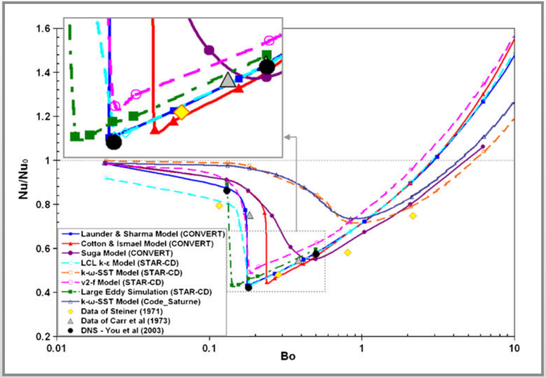
From Dehoux *et al.* (2017)

Vertical heated pipe of You *et al.* (2003)



From Keshmiri *et al.* (2012)

- ✓ Aiding buoyancy induces relaminarization (ascending flow case)
- ✓ Heat transfer in severely impaired



- ✓ Standard RANS models
- ✓ Relaminarization extremely sensitive to the model

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Algebraic flux models

$$\frac{\partial \overline{u_i\theta}}{\partial t} + U_k \frac{\partial \overline{u_i\theta}}{\partial x_k} = P_{i\theta}^U + P_{i\theta}^T + G_{i\theta} + \phi_{i\theta} - \varepsilon_{i\theta} + D_{i\theta}^{\nu,t,p}$$

✓ Weak equilibrium assumption: $\frac{\mathrm{d}}{\mathrm{d}t} \frac{\overline{u_i\theta}}{\sqrt{k}\sqrt{\overline{\theta^2}}} = 0$ and $\mathrm{Diff} \frac{\overline{u_i\theta}}{\sqrt{k}\sqrt{\overline{\theta^2}}} = 0$

$$P_{\theta i}^U + P_{\theta i}^T + G_{\theta i} + \phi_{\theta i}^* - \varepsilon_{\theta i} - \frac{\overline{u_i\theta}}{2k}\left(P_k + G_k - \varepsilon\right) - \frac{\overline{u_i\theta}}{2\overline{\theta^2}}\left(P_{\overline{\theta^2}} - \varepsilon_{\overline{\theta^2}}\right) = 0$$

✓ Equilibrium assumption: $P_k + G_k = \varepsilon$ and $P_{\overline{\theta^2}} = \varepsilon_{\overline{\theta^2}}$

$$\overline{u_i\theta} = -C_\theta \frac{k}{\varepsilon}\left[\zeta \overline{u_i u_j} \frac{\partial T}{\partial x_j} + \xi \overline{u_j\theta} \frac{\partial U_i}{\partial x_j} + \eta \beta g_i \overline{\theta^2}\right]$$

✓ The main physical mechanisms are present:

- ▷ The 3 production terms
- ▷ The 3 redistribution terms
- ▷ Near-wall effects can be included: EB-DFM → EB-AFM (Dehoux *et al.*, 2012)

Eddy-viscosity models

Boussinesq relation: $$\overline{u_i u_j} = -2\nu_t S_{ij} + \frac{2}{3} k \delta_{ij}$$

SGDH: $$\overline{u_i \theta} = -\frac{\nu_t}{Pr_t} \frac{\partial T}{\partial x_i}$$

✓ Production term $G_k = -\beta g_i \overline{u_i \theta} = \beta g_i \dfrac{\nu_t}{Pr_t} \dfrac{\partial T}{\partial x_i}$

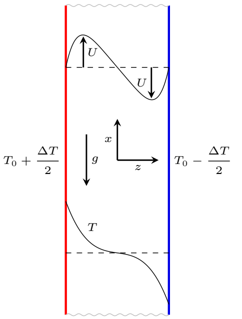
✓ Unstratified (or weakly stratified) flows:

▷ $g_i \dfrac{\partial T}{\partial x_i} = 0$ (orthogonal) $\Rightarrow G_k = 0$

$\Rightarrow$ G_ε as well

✓ Idea: SGDH in the temperature equation, GGDH in the production terms (Ince & Launder, 1987)

$$\begin{aligned} G_k &= -\beta g_i \overline{u_i \theta} = \beta g_i C \overline{u_i u_j} \frac{k}{\varepsilon} \frac{\partial T}{\partial x_i} \\ &= \beta g_i \frac{2}{3} C \frac{k^2}{\varepsilon} \frac{\partial T}{\partial x_i} - \beta g_i 2C \nu_t \frac{k}{\varepsilon} S_{ik} \frac{\partial T}{\partial x_k} \end{aligned}$$



Buoyancy-extended Eddy-Viscosity Models

$$\frac{\mathrm{D}\overline{u_i u_j}}{\mathrm{D}t} = P_{ij} + G_{ij} + \phi_{ij} - \varepsilon_{ij} + D_{ij}^{\nu,t,p}$$

✓ Weak equilibrium + Equilibrium ($P_k + G_k = \varepsilon$):

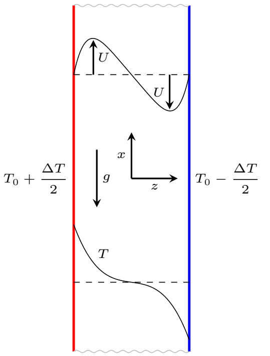
$$\overline{u_i u_j} = \underbrace{\frac{2}{3}k\delta_{ij} + \frac{k}{\varepsilon}\frac{1-C_2}{C_1}\left(P_{ij} - \frac{2}{3}P_k\delta_{ij}\right)}_{\text{Boussinesq part}} + \underbrace{\frac{k}{\varepsilon}\frac{1-C_3}{C_1}\left(G_{ij} - \frac{2}{3}G_k\delta_{ij}\right)}_{\text{Buoyancy extension}}$$

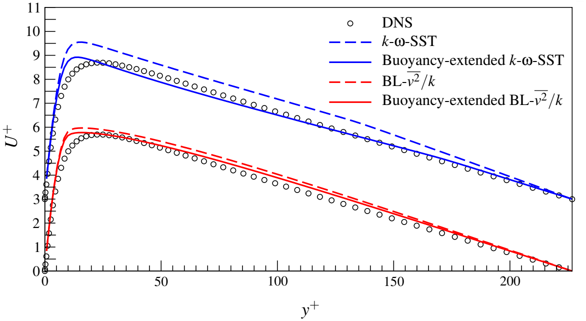
$$\Rightarrow \overline{u_i u_j} = \underbrace{\frac{2}{3}k\delta_{ij} - 2\nu_t S_{ij}}_{\overline{u_i u_j}_{\text{Bouss}}} + \underbrace{C_\theta^* \tau\left(G_{ij} - \frac{2}{3}G_k\delta_{ij}\right)}_{\overline{u_i u_j}_{\text{Buo}}} \qquad \text{(Davidson, 1990)}$$

✓ Associated with GGDH: $\overline{u_i\theta} = -C_\theta \tau\, \overline{u_i u_j}_{\text{Bouss}} \dfrac{\partial T}{\partial x_j} - C_\theta \tau\, \overline{u_i u_j}_{\text{Buo}} \dfrac{\partial T}{\partial x_j}$

✓ Automatically extends all the terms involving $\overline{u_i u_j}$ and $\overline{u_i\theta}$: P_k, G_k, G_ε

✓ Does not modify the model for forced convection





From Jameel *et al.* (2019)

- ✓ Balance of forces: $$\int_0^y \beta g(T - T_{ref})\mathrm{d}Y = \rho {u_\tau}^2 - \nu \frac{\partial U}{\partial y} + \overline{uv}$$

- ✓ Underestimation of $\overline{uv}$ $\Rightarrow$ overestimation of the mean velocity

- ✓ Contribution of the extension: $$\overline{uv} = -\nu_t \frac{\partial U}{\partial y} + C_\theta^* \tau \beta g \overline{v\theta}$$

Variable turbulent Prandtl number?

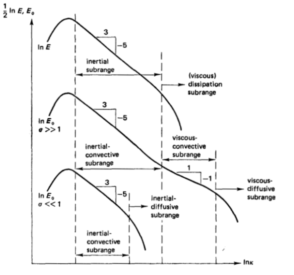


Figure 8.11. Spectra of temperature variance in liquids with large and small Prandtl numbers.

From Tennekes & Lumley (1972)

✓ Diffusion is due to mixing by large scales

✓ The same scales for mechanical and thermal turbulence

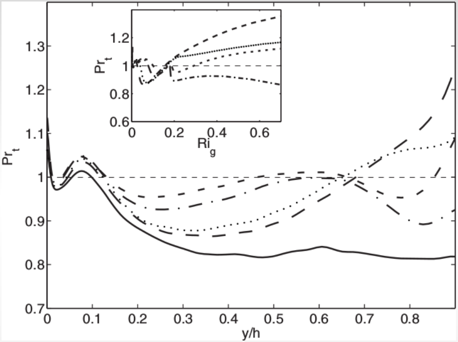
$\Rightarrow$ $Pr_t = \dfrac{\nu_t}{\kappa_t}$ must be close to unity

✓ Modifying Pr_t for buoyant flows is a common practice (atmosphere/ocean)

✓ Why?

▷ Should have a buoyancy extension: $\overline{v\theta} = -\dfrac{\nu_t}{Pr_t}\dfrac{\partial T}{\partial y} + \overline{v\theta}_{\text{Buo}}$

▷ To compensate: $-\dfrac{\nu_t}{Pr_t^*}\dfrac{\partial T}{\partial y} = -\dfrac{\nu_t}{Pr_t}\dfrac{\partial T}{\partial y} + \overline{v\theta}_{\text{Buo}} \Rightarrow Pr_t^* = \left(\dfrac{1}{Pr_t} + \dfrac{1}{\nu_t}\dfrac{\overline{v\theta}_{\text{Buo}}}{\partial T/\partial y}\right)^{-1}$



Pr_t extracted from a DNS of stably-stratified channel flow

From García-Villalba & del Álamo (2011)

✓ Modifying Pr_t is a patch

✓ Not a constant value but a function of the Richardson number

✓ Variations must be modest

Outline of the presentation

- ✓ **Introduction**
  - ▷ Influence of buoyancy in the equations of motion
  - ▷ Convection regimes
- ✓ **Influence of buoyancy on turbulence: physics → modelling**
  - ▷ Dynamics: energy, anisotropy, redistribution, dissipation
  - ▷ Heat fluxes
- ✓ **The devil is in the detail**
  - ▷ Dissipation
  - ▷ Time scales
  - ▷ Transition/Relaminarization
- ✓ **Simplified models?**
  - ▷ Algebraic flux models
  - ▷ Eddy-viscosity models
  - ▷ Variable Pr_t?
- ✓ **Unsteady approaches**
- ✓ **Conclusion**

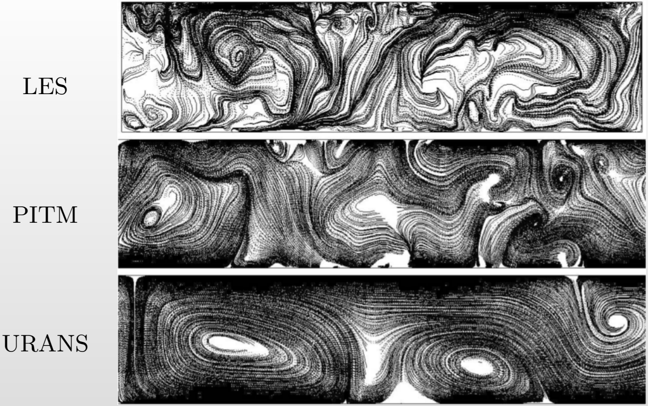
Unsteady approaches

Just a word

✓ URANS, LES or hybrid RANS/LES: $u_i^* = \underbrace{U_i + \widetilde{u}_i}_{\text{resolved}} + \underbrace{u_i''}_{\text{unresolved}}$

✓ Buoyancy is directly accounted for in the resolved part of turbulence $\widetilde{u}_i$

⇒ the contribution of buoyancy to modelled scales is smaller



Rayleigh-Bénard convection at $Ra = \times 10^9$
From Kenjereš & Hanjalić (2006)

The MONACO_2025 project

http://monaco2025.gforge.inria.fr

✓ Tackle the industrial simulation of transient, turbulent flows affected by buoyancy effects

✓ Bring together

- ▷ Two academic partners: LMAP-University of Pau and Institute PPrime-Poitiers
  - Turbulence modelling
  - Experimental studies
- ▷ And R&D departments of two industrial partners:
  - Automobile: PSA group
  - Energy production: EDF

Objectives

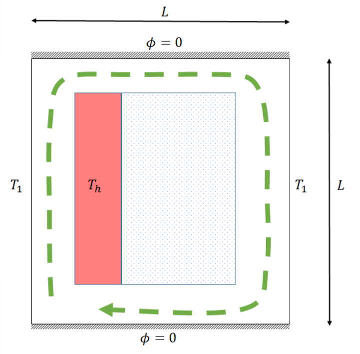
✓ **Scientific objective**

- ▷ Breakthrough in *the unresolved issue* of the modelling of turbulence/buoyancy interactions in transient situations
- ▷ Within the continuous hybrid RANS/LES paradigm
- ▷ Transient cavity flow experiments: *an unrivalled source of knowledge* for turbulence modelling

✓ **Industrial objective**

- ▷ *To make available computational methodologies* to address dimensioning, reliability and security issues in buoyancy-affected transient flows
- ▷ Problems are *not tackled using CFD at present in the industry*
- ▷ At the end:
  - Panel of methodologies (simple URANS $\rightarrow$ sophisticated hybrid model)
  - Evaluated in transient situations, against the dedicated cavity flow experiments and a real car underhood configuration
- ▷ *Decision-making tool* for the industrial partners
- ▷ In line with the *Full Digital 2025 ambition*

New experimental facility (Poitiers)



- ▷ $Ra \approx 10^{10}$
- ▷ Steady state + transient + cyclic
- ▷ TR-PIV + microthermocouples
- ▷ Reynolds stresses, temperature variance, wall heat flux

✓ Experimental database → ERCOFTAC Nexus and/or QNET-CFD databases

✓ ERCOFTAC SIG15 Workshop in Pau end of 2021

Thank you for your attention

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✓ **Variation of the physical properties :**
Approximated dependence laws can be used for the physical properties of the fluid.

- ▷ For instance, Sutherland's law is often used to describe the evolution of the viscosity with temperature:

$$\frac{\mu(T)}{\mu_0} = \left(\frac{T}{T_0}\right)^{3/2} \frac{T_0 + S}{T + S} \tag{2}$$

- ▷ The heat capacity C_p and the Prandtl number Pr are often considered constant, such that

$$\lambda(T) = \frac{\mu(T)C_p}{Pr} = \frac{\mu_0 C_p}{Pr}\left(\frac{T}{T_0}\right)^{3/2} \frac{T_0 + S}{T + S} \tag{3}$$

✓ Low-Mach-number approximation:

- For low Mach numbers, it can be assumed that density does not depend on pressure, but only on temperature

$$\rho^* = f(\cancel{P^*}, T^*)$$

- For a perfect gas, we have

$$p^* = \rho^* r T^*$$

such that

$$\mathrm{d}\rho^* = \cancel{\frac{\partial \rho^*}{\partial P^*}}\bigg|_{T^*} \mathrm{d}P^* + \frac{\partial \rho^*}{\partial T^*}\bigg|_{P^*} \mathrm{d}T^* = -\frac{\rho^*}{T^*}\mathrm{d}T^*$$

and

$$\rho^* = \rho_0^* \frac{T_0^*}{T^*}$$

- The flow is thus considered incompressible, but density varies as a function of the inverse of the temperature: the fluid is dilatable.

- The equations of motion can be derived using asymptotic expansions at the limit of small Mach numbers.

Redistribution

✓ Chou's analysis is modified

$$\phi_{ij} = \underbrace{\phi_{ij}^1}_{\text{Slow term}} + \underbrace{\phi_{ij}^2}_{\text{Rapid term}} + \underbrace{\phi_{ij}^3}_{\text{Buoyant term}}$$

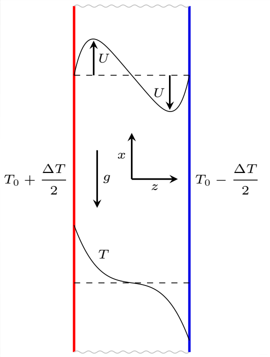
▷ Rapid term

Production: $P_{ij} = -\overline{u_i u_k}\dfrac{\partial U_j}{\partial x_k}$ IP model: $\phi_{ij}^2 = -C_2\left(P_{ij} - \dfrac{2}{3}P\delta_{ij}\right)$

▷ Buoyant term: similar

Production: $G_{ij} = -\beta g_i \overline{u_j \theta} - \beta g_j \overline{u_i \theta}$ IP model: $\phi_{ij}^3 = -C_3\left(G_{ij} - \dfrac{2}{3}G\delta_{ij}\right)$

Example: differentially heated vertical channel



✓ In this case, the production terms read:

Component	Strain production	T gradient production	Buoyancy production
$\overline{u\theta}$	$-\overline{w\theta}\frac{\partial U}{\partial z}$	$-\overline{uw}\frac{\partial T}{\partial z}$	$\beta g \overline{\theta^2}$
$\overline{v\theta}$	0	0	0
$\overline{w\theta}$	0	$-\overline{w^2}\frac{\partial T}{\partial z}$	0

✓ Again: particular direction = vertical direction

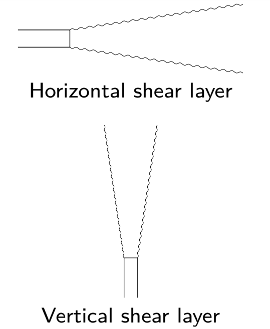
Flux Richardson number

✓ Originally: relative weight of buoyancy production

$$Ri = \frac{\beta g\, \Delta T\, L_{\text{ref}}}{U_{\text{ref}}^2} \qquad Ri_f = \frac{\beta g\, \overline{u\theta}}{-\overline{uw}\, \partial U/\partial z} \text{ (boundary layer, Bradshaw, 1969)}$$

$$\Rightarrow Ri_f = -\frac{G_k}{P_k}$$

✓ Rodi (1979): to differentiate vertical and horizontal stratified 2D shear layers



✓ G_ε necessary for an horizontal layer, not for a vertical layer

✓ $Ri_f = -\dfrac{1}{2}\dfrac{G_{\overline{v^2}}}{P_k + G_k}$ where $\overline{v^2}$ normal to the flow

✓ Horizontal: $\overline{v^2}$ aligned with g $\Rightarrow G_{\overline{v^2}} = 2\beta g \overline{v\theta} = 2G \;\Rightarrow\; Ri_f = -\dfrac{G_k}{P_k + G_k} \in [-1; 1]$

✓ Vertical: $G_{\overline{v^2}} = 0 \;\Rightarrow\; Ri_f = 0$

✓ $G_{\overline{v^2}}$ not a valid concept in 3D

$$\Rightarrow Ri_f = -\frac{G_k}{P_k + G_k} \text{ is used}$$