Imperial College London

Direct Numerical Simulation of turbulent natural convection in a square cavity

SIG15 – 16th ERCOFTAC Workshop
Wednesday, 16th October 2019

Dr Frederic Sebilleau
Department of Mechanical Engineering

Co-authors: *Dr R. I. Issa[1], Dr S. Walker[1], Dr Sylvain Lardeau[2]*

[1]Department of Mechanical Engineering, Imperial College London
[2]Siemens PLM Software, , Germany

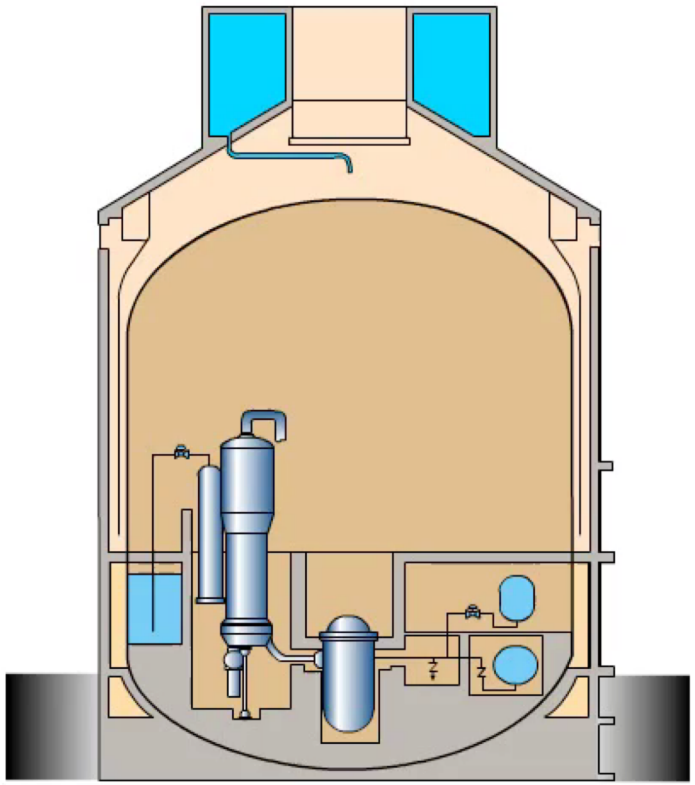
Content

- Context of the work
- Presentation of the DNS database
- Discussion on the DNS data
- EBRSM comparison against DNS
- A-priori analysis of EBRSM closure relations

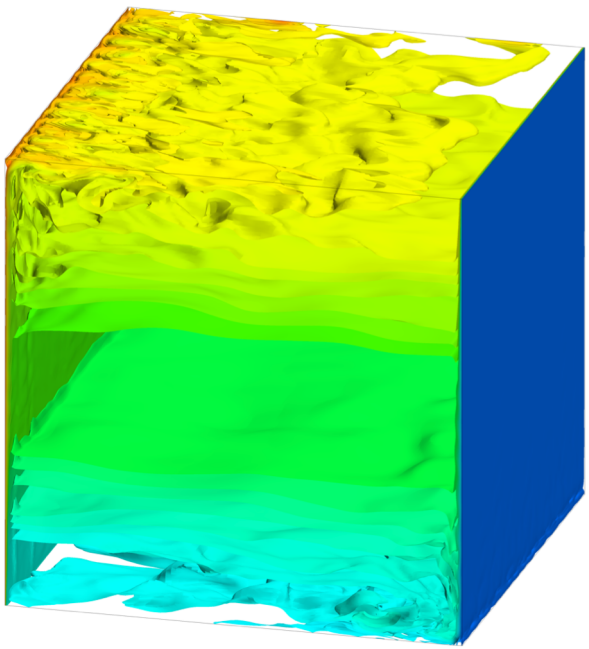
Context of the work

Context

What are the modelling requirements to predict reliably buoyancy driven flows in nuclear reactor configurations ?



Westinghouse AP 1000 containment passive cooling system



Differentially heated square cavity

Imperial College London

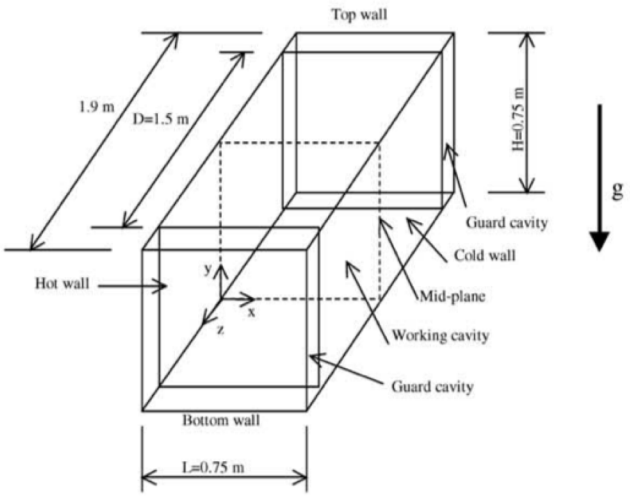
(Some) existing work on differentially heated cavities

Author	Year	Ra	Pr	Boussinesq ?	Horizontal BC	AR_x	AR_z	N_{dim}	Type
Paolucci	1990	1.0×10^{10}	0.71	No	adiabatic	1	0	Full 2D	DNS
Dafa'Alla and Betts	1996	0.83×10^{6}	0.71	Yes	adiabatic	28.6	7	2D stats	exp
Le Quéré and Benia	1998	1.0×10^{10}	0.71	Yes	Adiabatic	1	0	Full 2D	DNS
Betts and Bokhari	2000	1.43×10^{6}	0.71	Yes	adiabatic	28.6	7	2D stats	exp
Tian and Karayanis	2000	1.58×10^{9}	0.71	Yes	Measured profile	1	2	2D stats	exp
Ampofo and Karayanis	2003	1.58×10^{9}	0.71	Yes	Measured profile	1	2	2D stats	exp
Salat	2004	1.5×10^{9}	0.71	Yes	Measured profile	1	0.32	Full 3D	exp
Soria et al.	2004	6.4×10^{8}	0.71	Yes	Adiabatic et PCW	4	1	2D stats	DNS
Baghardi and Davidson	2007	4.03×10^{8}	0.71	Yes	Adiabatic	5	0.4	2D stats	LES
Trias et al.	2010	1.0×10^{11}*	0.71	Yes	Adiabatic	4	0.25	2D stats	DNS
Puragliesi et al.	2011	1.0×10^{10}	0.71	Yes + particles	Adiabatic	1	0	Full 2D	DNS
Saury et al.	2011	1.2×10^{11}	0.71	Yes	Adiabatic	3.84	0.86	3D	exp
Puragliesi et al.	2012	1.0×10^{9}	0.71	Yes	Adiabatic	1	1	3D	DNS
Lau et al.	2013	4.56×10^{10}	0.71	No	Adiabatic	5	2	3D	LES
Sergent et al.	2013	1.5×10^{9}	0.71	Yes	Measured profile (Salat)	1	0.32	3D	DNS
Kizildag et al.	2014	3.0×10^{11}*	4.31	Yes	Adiabatic	5	1	2D stats	DNS

**Ra based on cavity height*

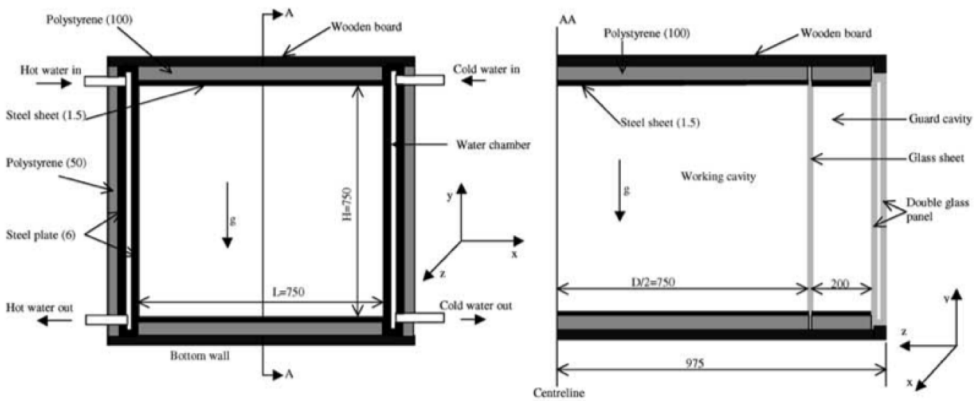
The starting point: experimental data of Ampofo et al. (1/2)

Experimental cavity parameters (Ampofo et al. (2003))



Parameters:
$T_h = 50\ ^oC$
$T_c = 10\ ^oC$
$Ra = 1.58 \times 10^9$
$AR_x = 1$
$AR_z = 2$
$Pr = 0.71$

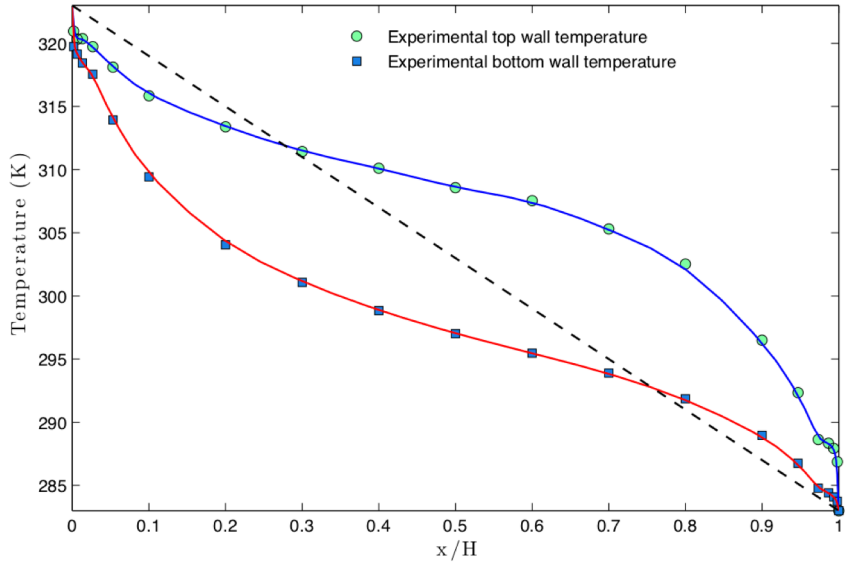
(b)



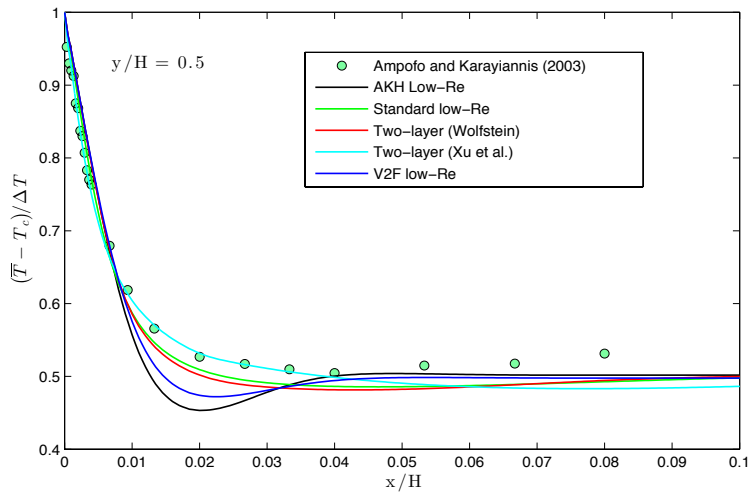
Precision of the experimental data

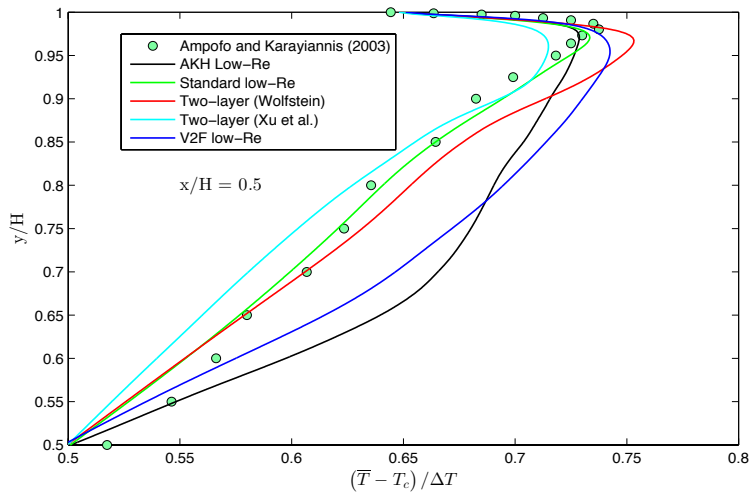
Parameter	Degree of uncertainty
Wall temperature	0.15 K
Air temperature	0.10 K
Air velocity	0.07%
Reynolds stress	0.10%
Turbulent heat flux	0.15%
Rayleigh number	0.62%
Nusselt number	0.25–1.13%
Wall shear stress	1.38%

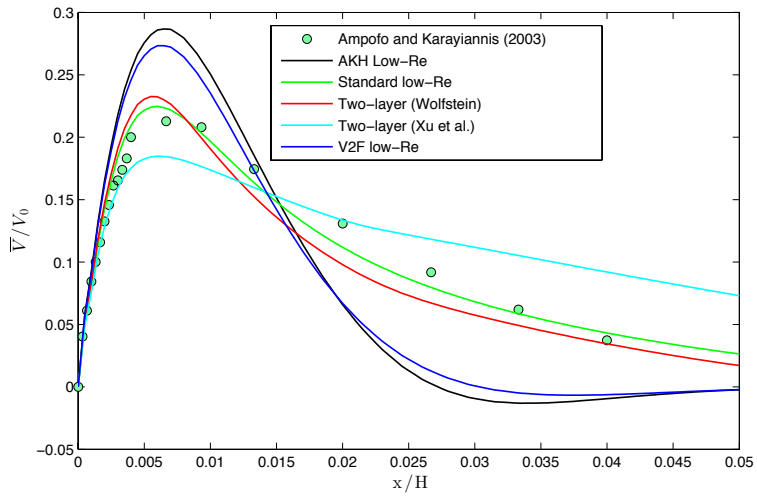
Horizontal walls boundary condition



The starting point: experimental data of Ampofo et al. (2/4)







CFD code: STAR-CCM+

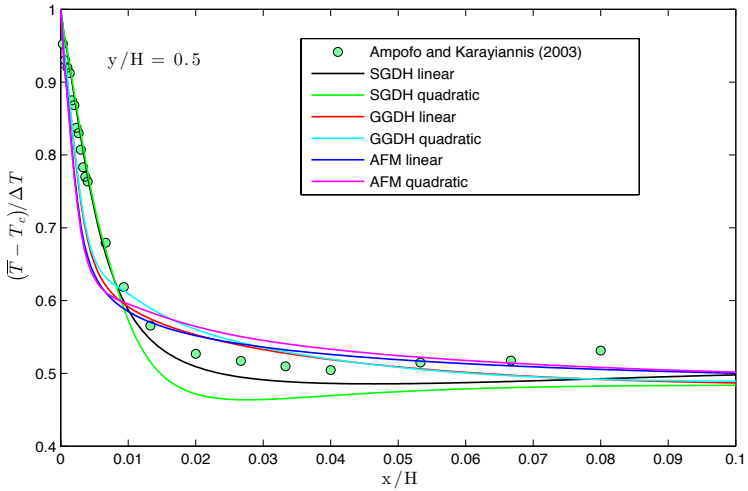
Domain: 2D

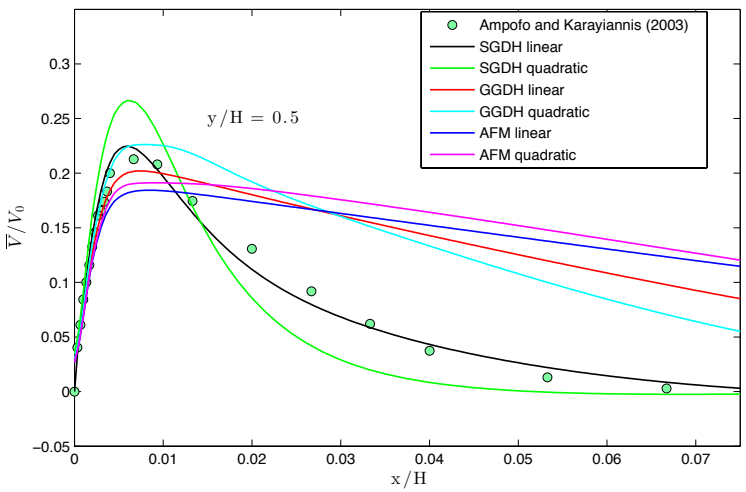
Simulation type: RANS (linear eddy viscosity model)

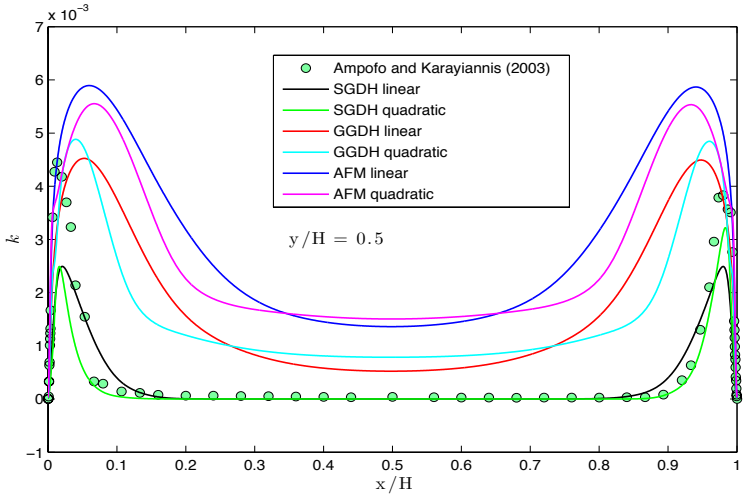
Turbulent heat flux treatment: SGDH

☛ Very large discrepancies between the models around the experimental data

The starting point: experimental data of Ampofo et al. (3/4)







CFD code: STAR-CCM+

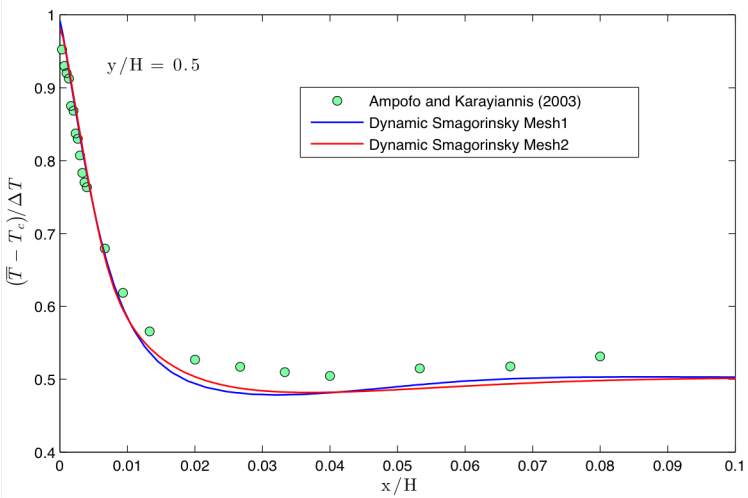
Domain: 2D

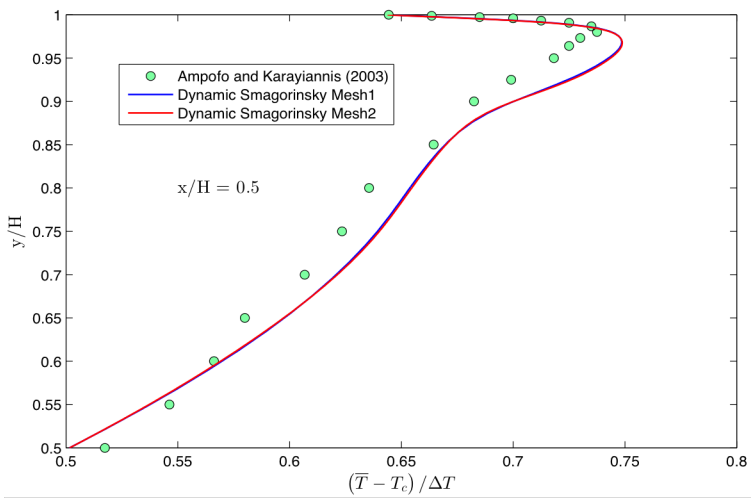
Simulation type: linear and quadratic Lien low-Re k-eps

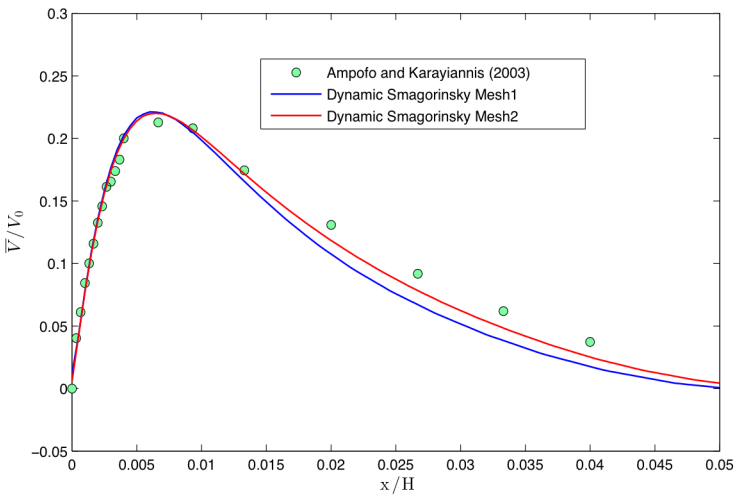
Turbulent heat flux treatment: SGDH, GGDH, AFM

☛ Better turbulent heat flux formulation causes larger discrepancies wrt experimental data

The starting point: experimental data of Ampofo et al. (4/4)







CFD code: STAR-CCM+

Domain: 3D

Simulation type: LES (dynamic Smagorinsky SGS)

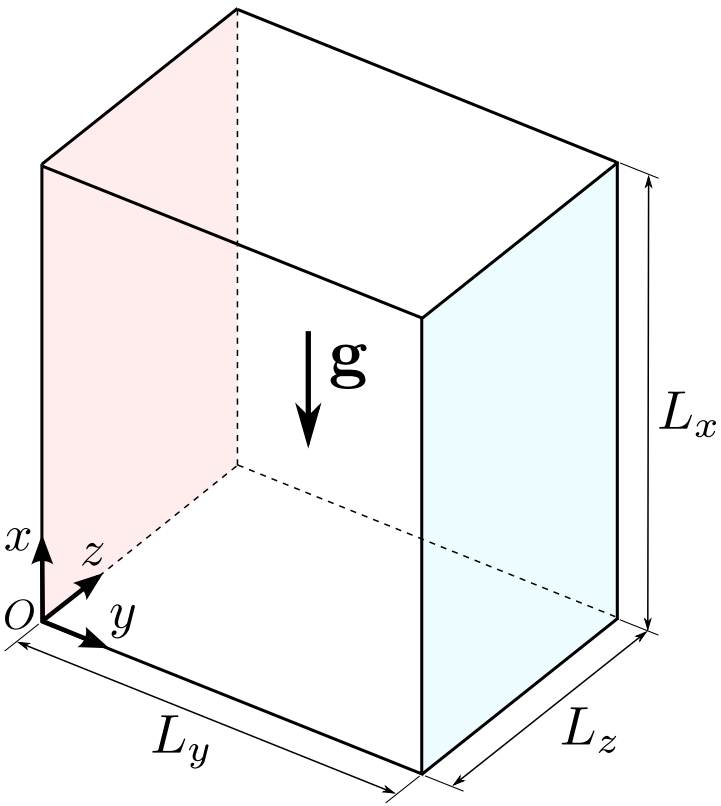
Turbulent heat flux treatment: SGDH

☛ LES statistics is not better than RANS to match experimental data

Presentation of the DNS database

Computational domain and governing equations

Computational domain



$L_x = L_y = H$

L_z / H depends on Ra

Governing equations (adimensionalised as solved)

$$\frac{\partial u_j}{\partial x_j} = 0$$

$$\frac{\partial u_i}{\partial t} + \frac{1}{2}\left(\frac{\partial u_i u_j}{\partial x_j} + u_j \frac{\partial u_i}{\partial x_j}\right) = -\frac{\partial p}{\partial x_i} + PrRa^{-1/2}\frac{\partial^2 u_i}{\partial x_j \partial x_j} - \Pr\theta \frac{g_i}{\mathbf{g}}$$

$$\frac{\partial \theta}{\partial t} + u_j \frac{\partial \theta}{\partial x_j} = Ra^{-1/2}\frac{\partial^2 \theta}{\partial x_j \partial x_j}$$

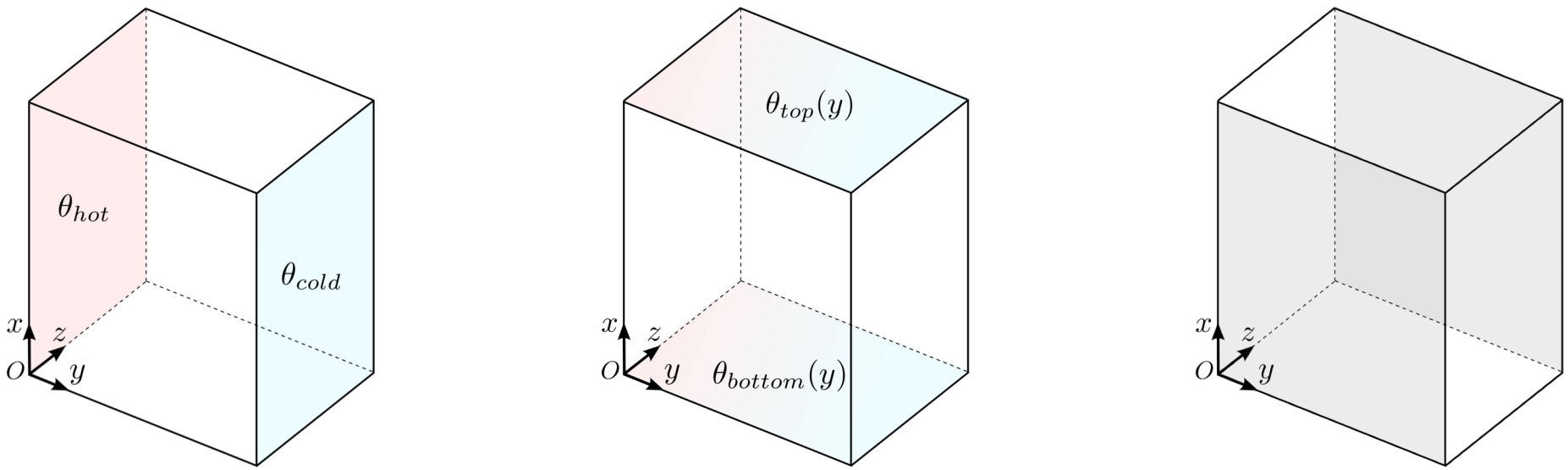
Where:

$$\mathbf{u}^\star = \frac{\alpha Ra^{1/2}}{H}\mathbf{u} \qquad p^\star = \frac{\alpha^2 Ra}{H^2}p \qquad \theta^\star = \theta\Delta\theta^\star + \theta^\star_{avg}$$

$$t^\star = \frac{H^2 Ra^{-1/2}}{\alpha}t \qquad \mathbf{x}^\star = H\mathbf{x} \qquad Ra = \frac{g\beta\Delta\theta^\star H^3}{\nu\alpha} \qquad Pr = \frac{\nu}{\alpha}$$

Starred quantities denote the corresponding physical quantity

Boundary conditions



Side walls

- No-slip condition
- Prescribed uniform temperatures:

$$\theta_{cold} = -0.5$$

$$\theta_{hot} = +0.5$$

Horizontal walls

- No-slip condition
- Prescribed temperature profiles:
  - ☛ *Either linear:*

$$\theta_{top}(y) = \theta_{bottom}(y) = 0.5 - y$$

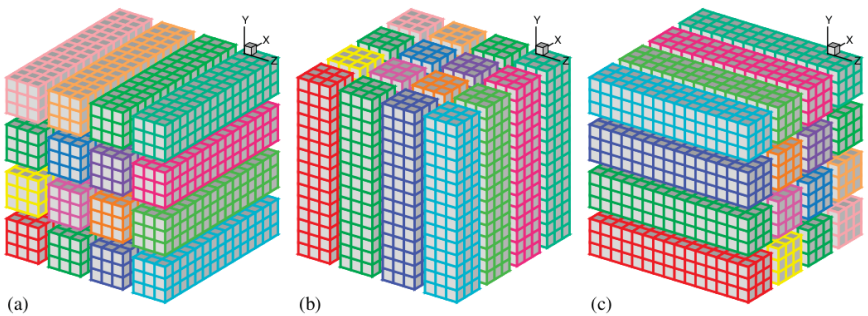
  - ☛ *Or interpolated experimental profile*

Front and back walls

- Periodic BC for all variables

Brief presentation of the DNS code: Incompact3D

- Cartesian 3D grid (N_x - N_y - N_z)
- 6th order compact finite difference scheme
- Scalable 2D domain decomposition library



- Pressure velocity coupling: three stage fractional step method
- Time stepping: fully explicit (3rd order Runge-Kutta scheme)
- Staggered grid for pressure
- Spectral pressure solver
- Non-uniform mesh only along *y*-direction:

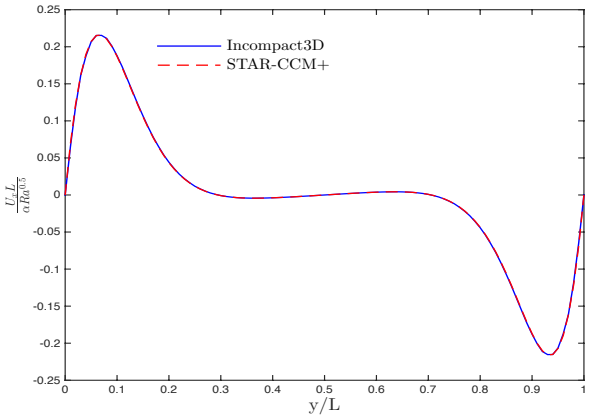
☛ Limitation originating from the direct solver in spectral space for pressure

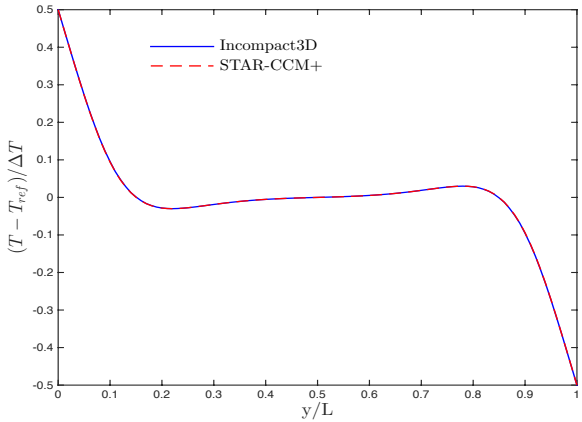
☛ *Implies a large number of cells in the x-direction, limiting the Ra reachable*

Incompact3D developments (1/2)

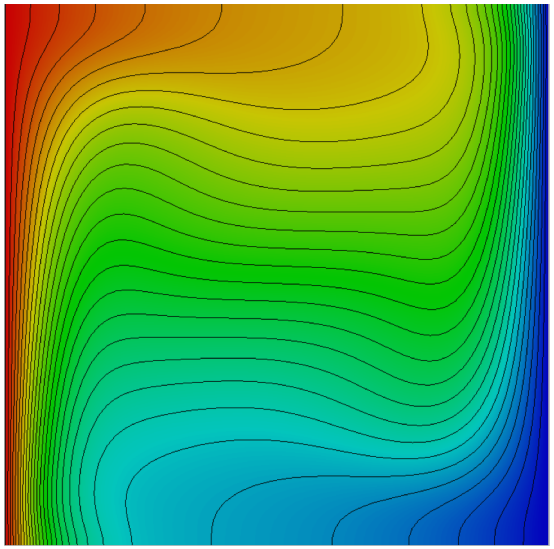
- Addition of the buoyancy source term in momentum equation
- Governing equations recast in previously detailed non-dimensional form

☛ Tested against the commercial code STAR-CCM+ for laminar adiabatic cavity at Ra = 10^5:

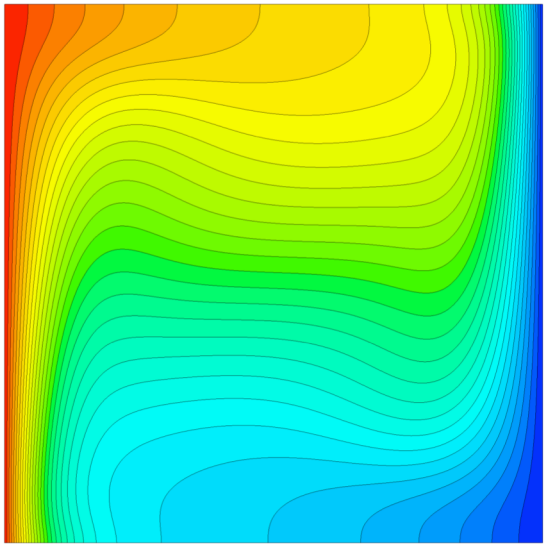




Incompact3D



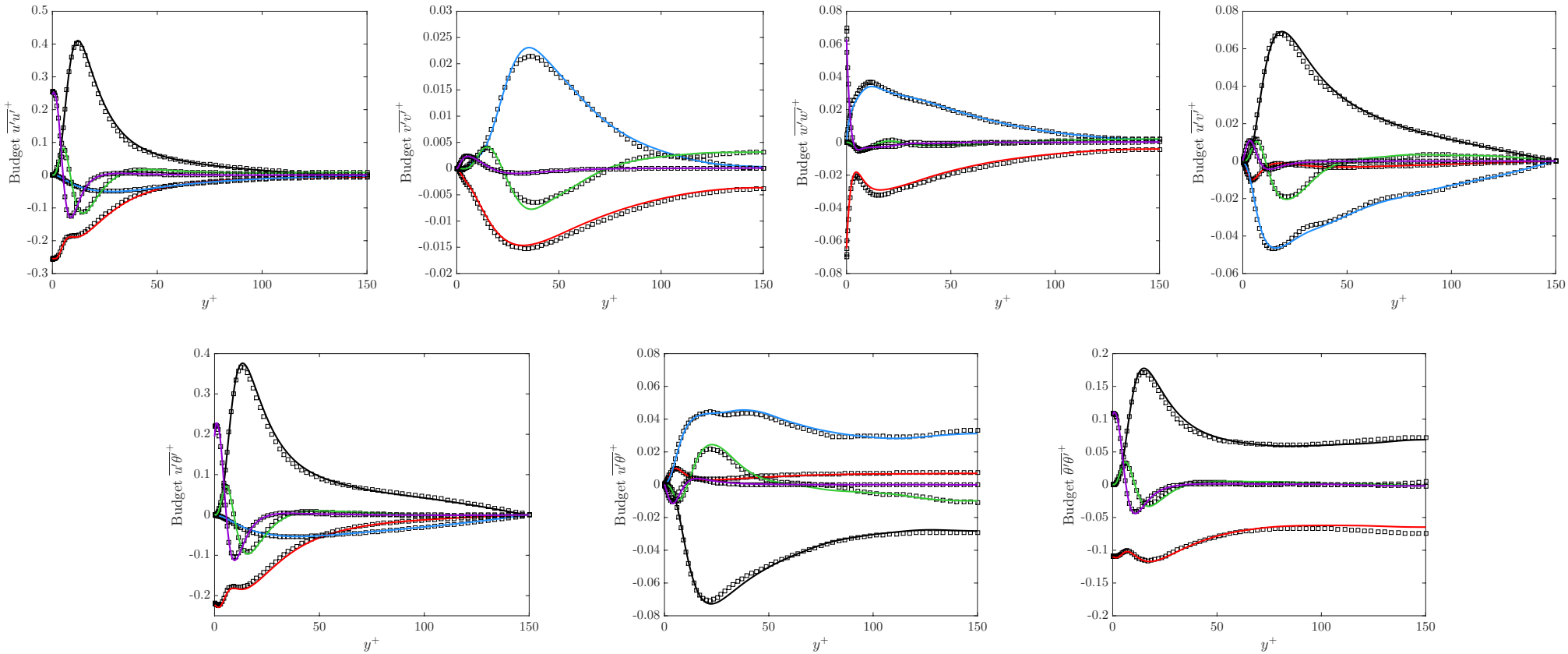
Star-CCM+



Imperial College London

Incompact3D developments (2/2)

- Implementation of machine accuracy restart procedure for postprocessing variables
- Implementation of full second order moments budgets
  - ☛ Tested against the passive scalar channel flow DNS of Kasagi et al. at $Re_\tau = 150$



Imperial College London

Overview of the available DNS simulations

Simulation	Ra	L_z/H	N_x	N_y	N_z	Δt	Horizontal BC	Δt_{dev}	Δt_{avg}
DNS1	1.0×10^{8}	1.0	513	257	128	7.0×10^{-4}	Linear	200	350
DNS2	1.58×10^{9}	0.8	513	513	128	1.0×10^{-3}	Exp. profile	100	125
DNS3	1.58×10^{9}	0.4	1025	1025	64	2.5×10^{-4}	Exp. profile	20	100
DNS4	1.58×10^{9}	0.4	1025	513	128	7.0×10^{-4}	Exp. profile	20	160
DNS5	1.58×10^{9}	0.4	1025	513	128	7.0×10^{-4}	Linear	50	150
DNS6	1.0×10^{10}	0.3	2049	1025	128	3.0×10^{-4}	Linear	50	120
DNS7	1.0×10^{11}	0.15	2593	1459	192	2.0×10^{-4}	Linear	25	55

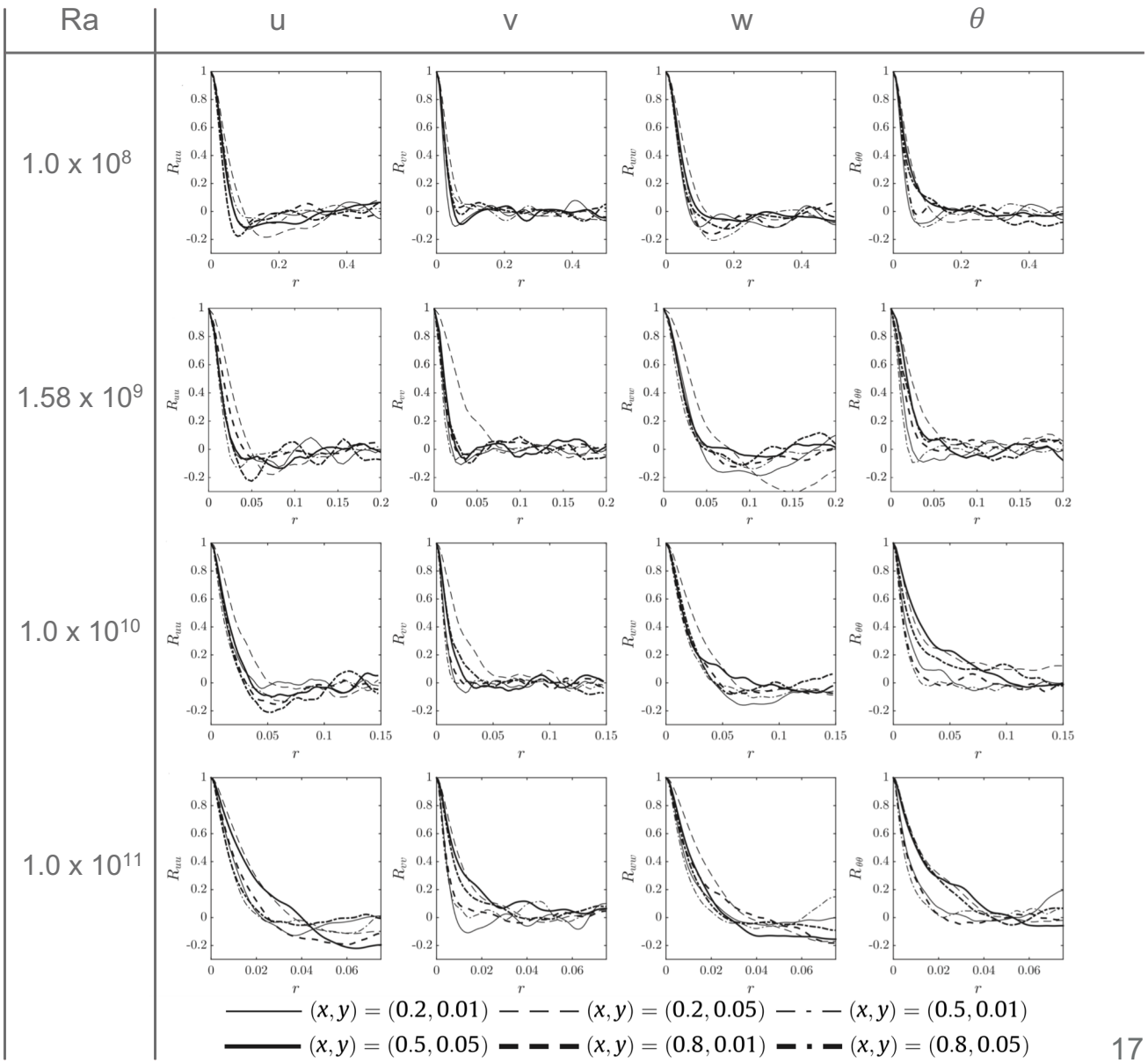
Choice of domain size and statistical averaging

For each quantity ϕ(x,y,z,t), the corresponding averaged quantity is defined by:

$$\bar{\phi}(x,y)=\frac{1}{\Delta t_{avg}}\frac{1}{L_z}\int_z\int_t \phi(x,y,z,t)dtdz$$

For each quantity ϕ(x,y,z,t), the two-point correlation along the z-direction $R_{\phi\phi}$ must decay sufficiently:

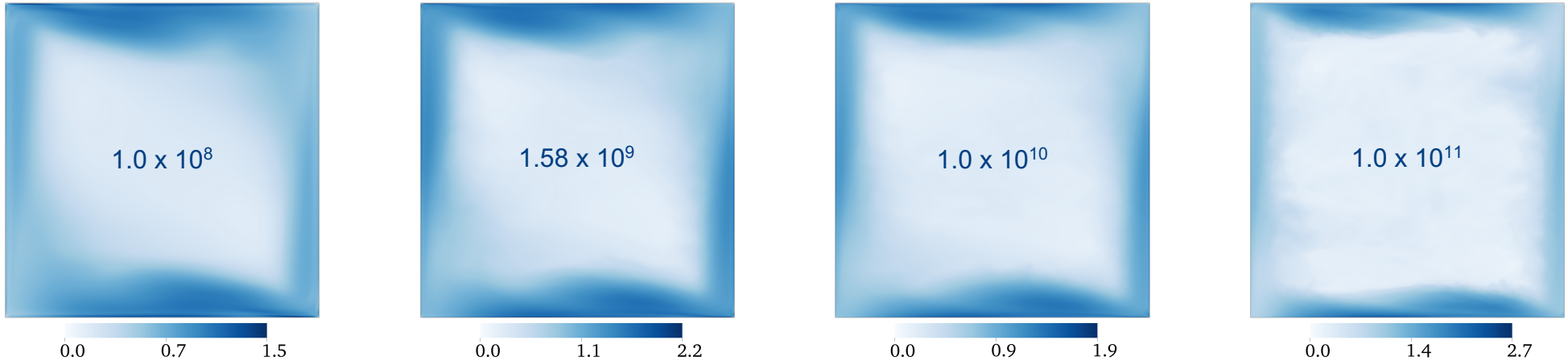
$$R_{\phi\phi}\left(x,y,r\right)=\frac{\overline{\phi'(x,y,z,t)\phi'(x,y,z+r,t)}}{\overline{\phi'(x,y,z,t)^2}}$$



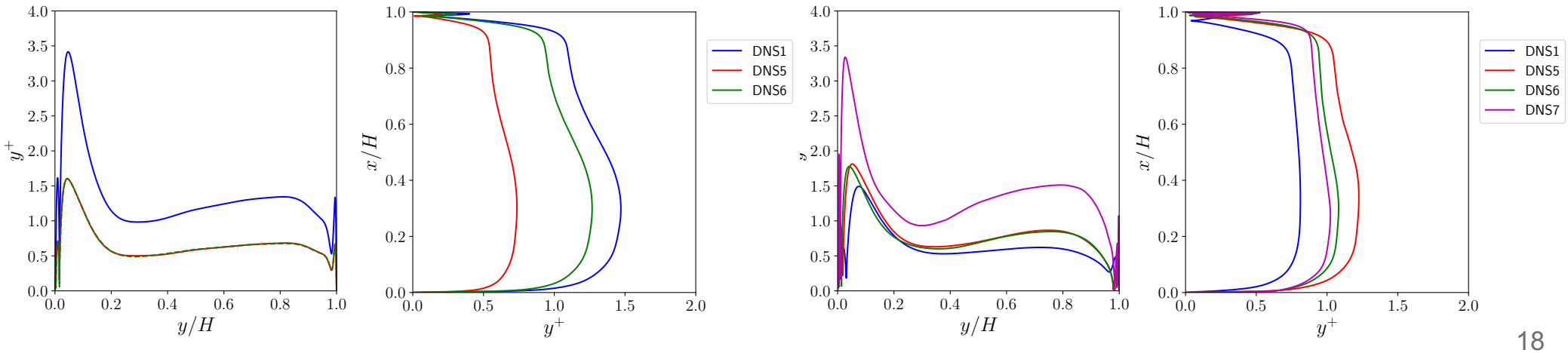
Imperial College London

Mesh refinement

Max Kolmogorov to gridspace : $\left(\Delta x \Delta y \Delta z\right)^{1/3} / \eta \sim 2-3$



Wall y+ around 1 on vertical boundary layer and trade off for top and bottom walls:

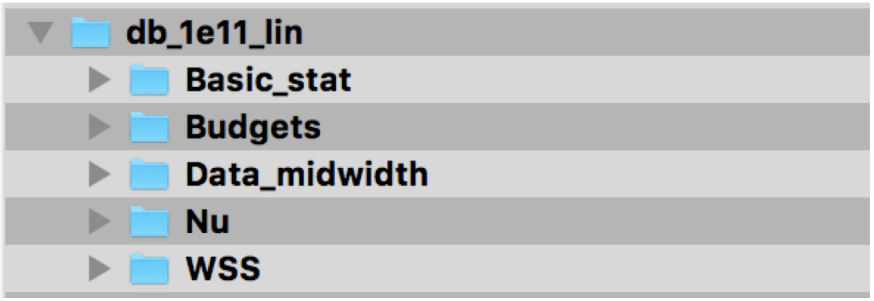


Imperial College London

Data available in the database

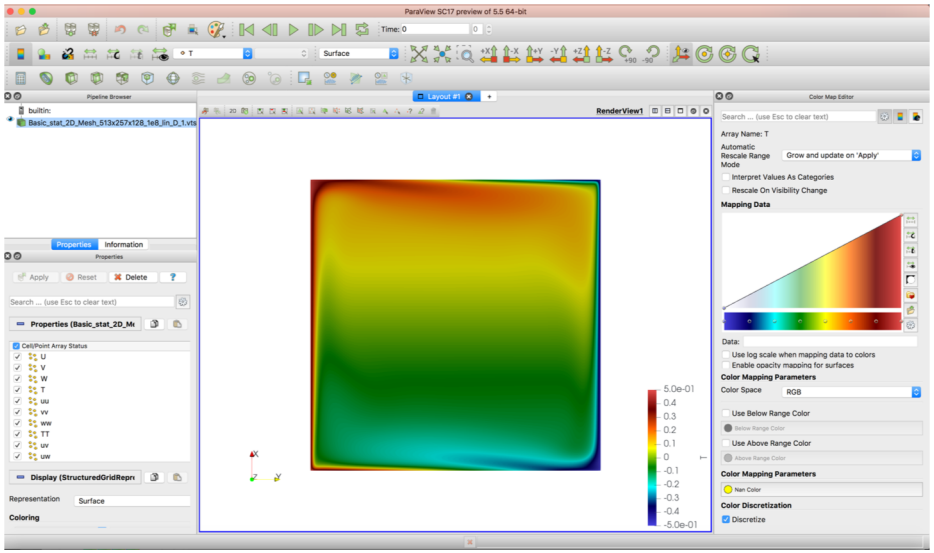
Data included in the ERCOFTAC archive:

- Basic stats extracted along several x/H isolines
- Budgets extracted along same isolines
- Basic stats along the line y/H = 0.5
- Nusselt on all four walls
- Wall shear stress on all four walls



Additional data available:

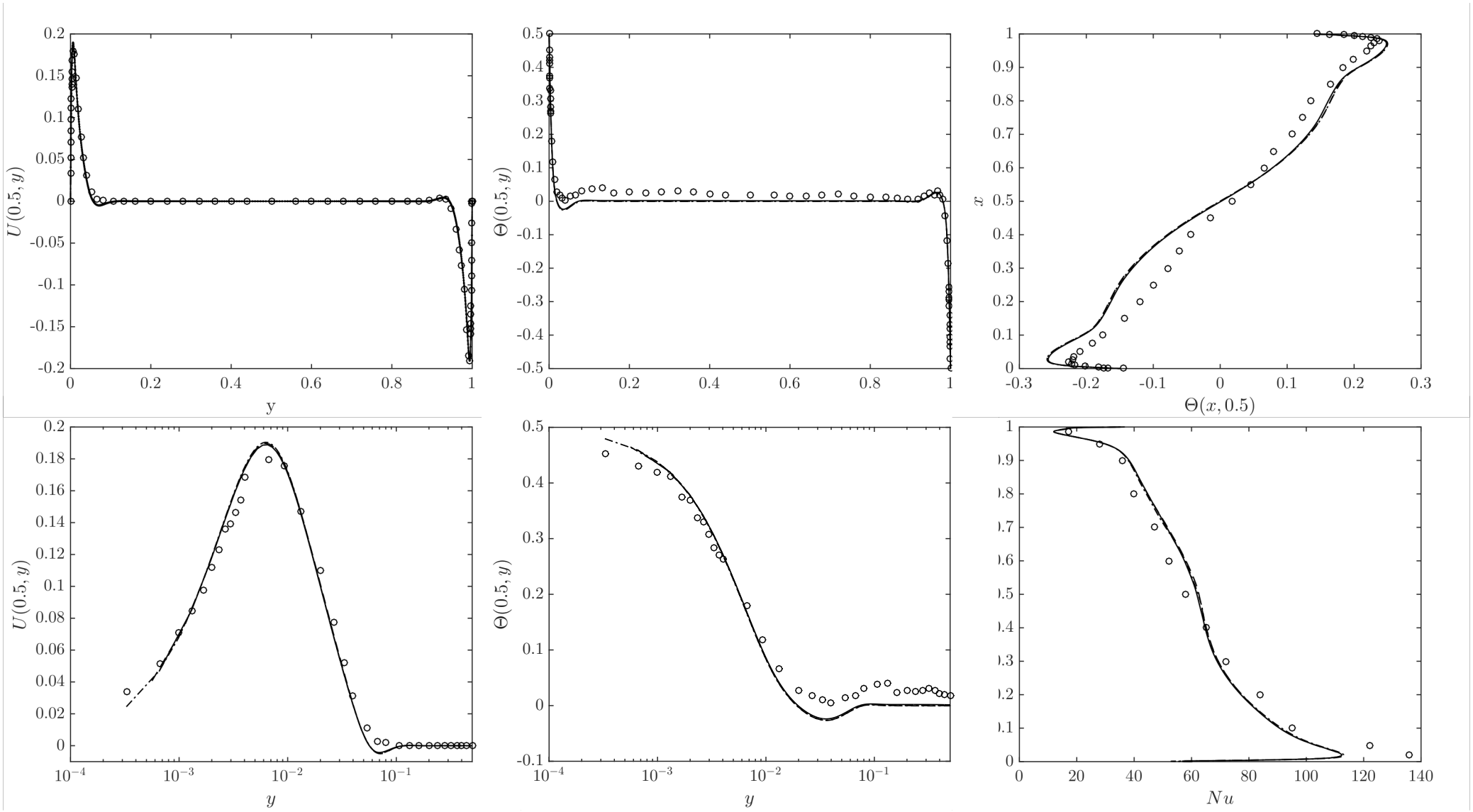
- 2D VTK files containing all the statistics at every location in the cavity
- 2D raw averaged arrays (those used to compute the budgets and first and second order moments
- 3D snapshot of velocity and temperature
- 2D snapshots at z = $L_z/2$ and x/H = 0.2 – 0.5 – 0.8



Analysis of the data

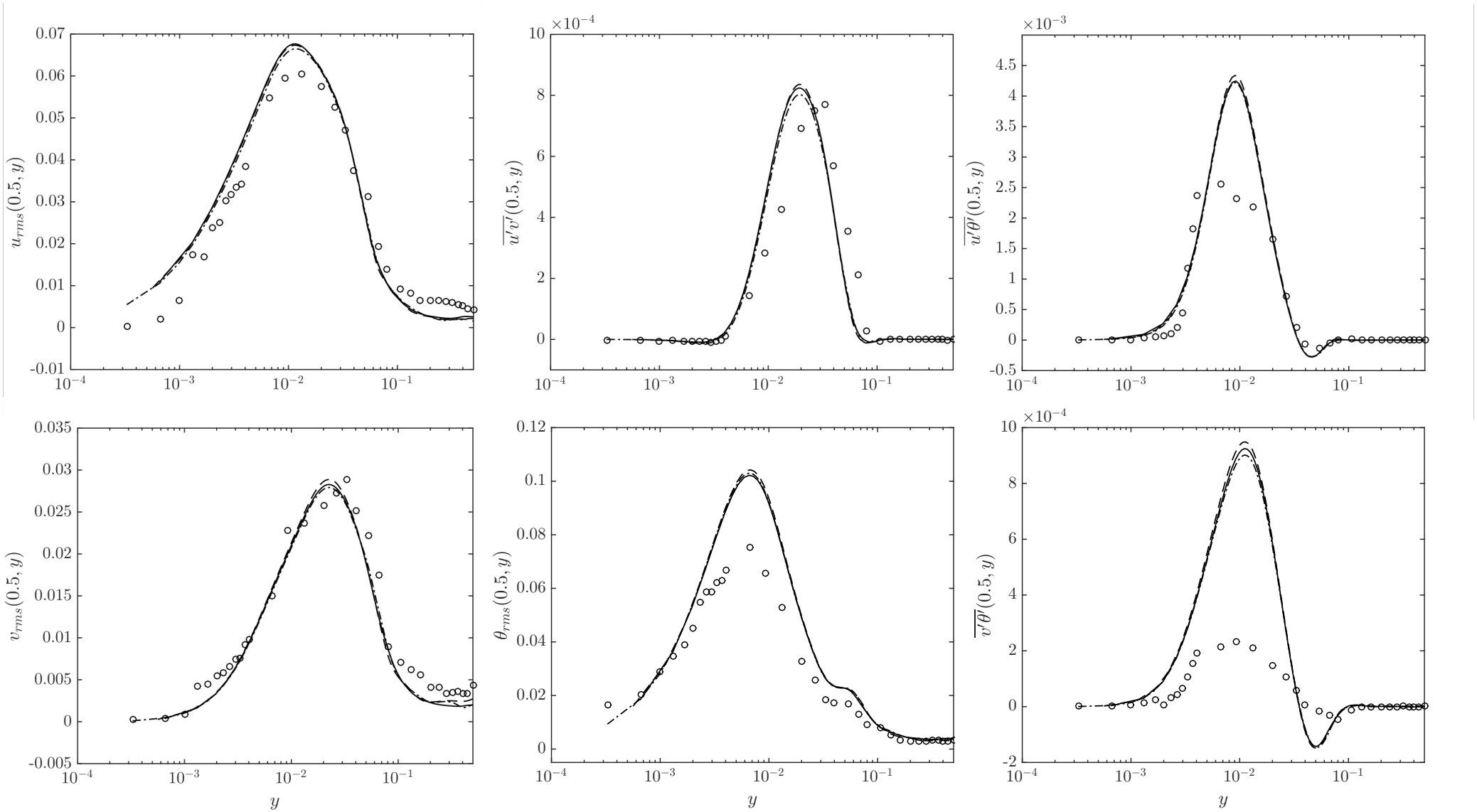
Imperial College London

Ampofo's cavity: Mesh sensitivity and comparison (1/2)



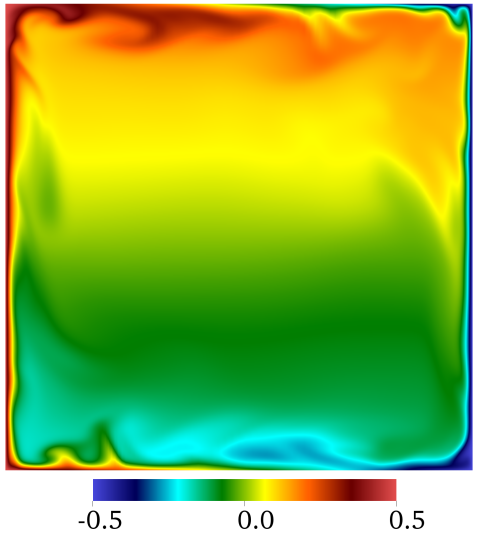
Imperial College London

Ampofo's cavity: Mesh sensitivity and comparison (2/2)

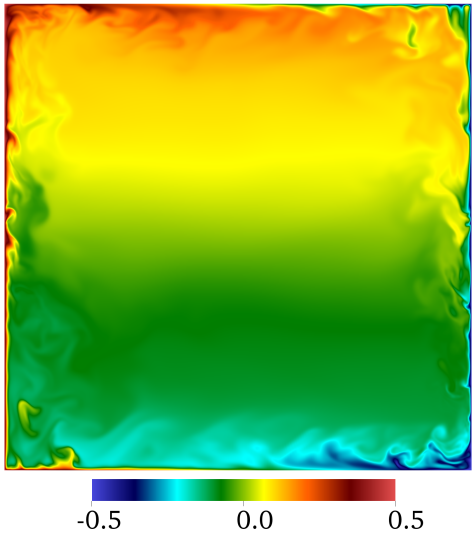


Flow structures: temperature snapshots

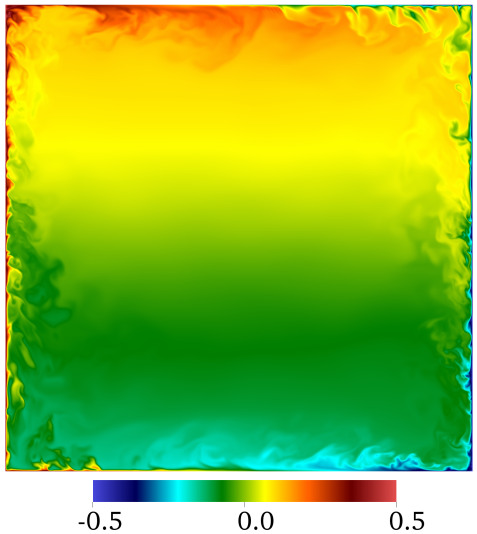
1.0×10^{8}



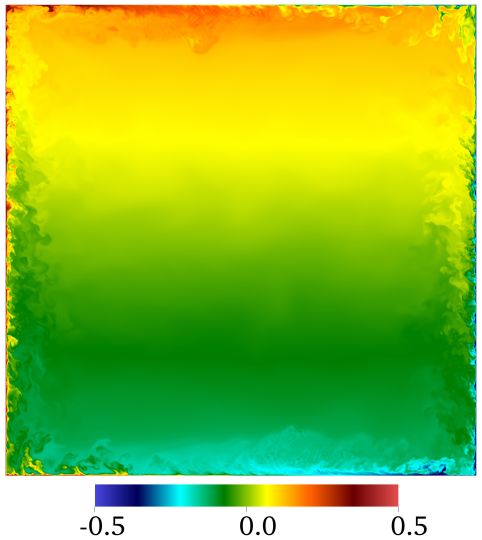
1.58×10^{9}



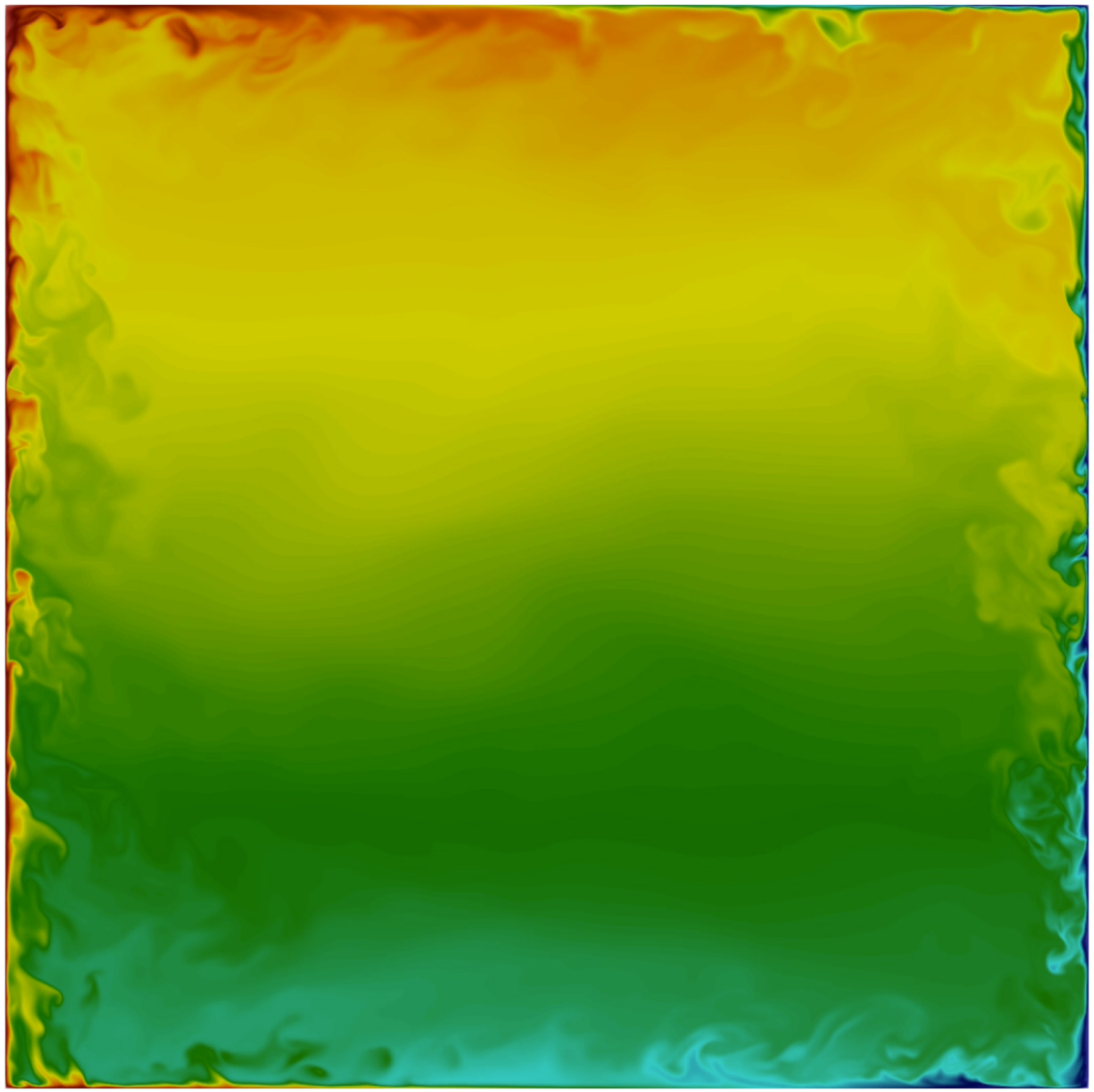
1.0×10^{10}



1.0×10^{11}



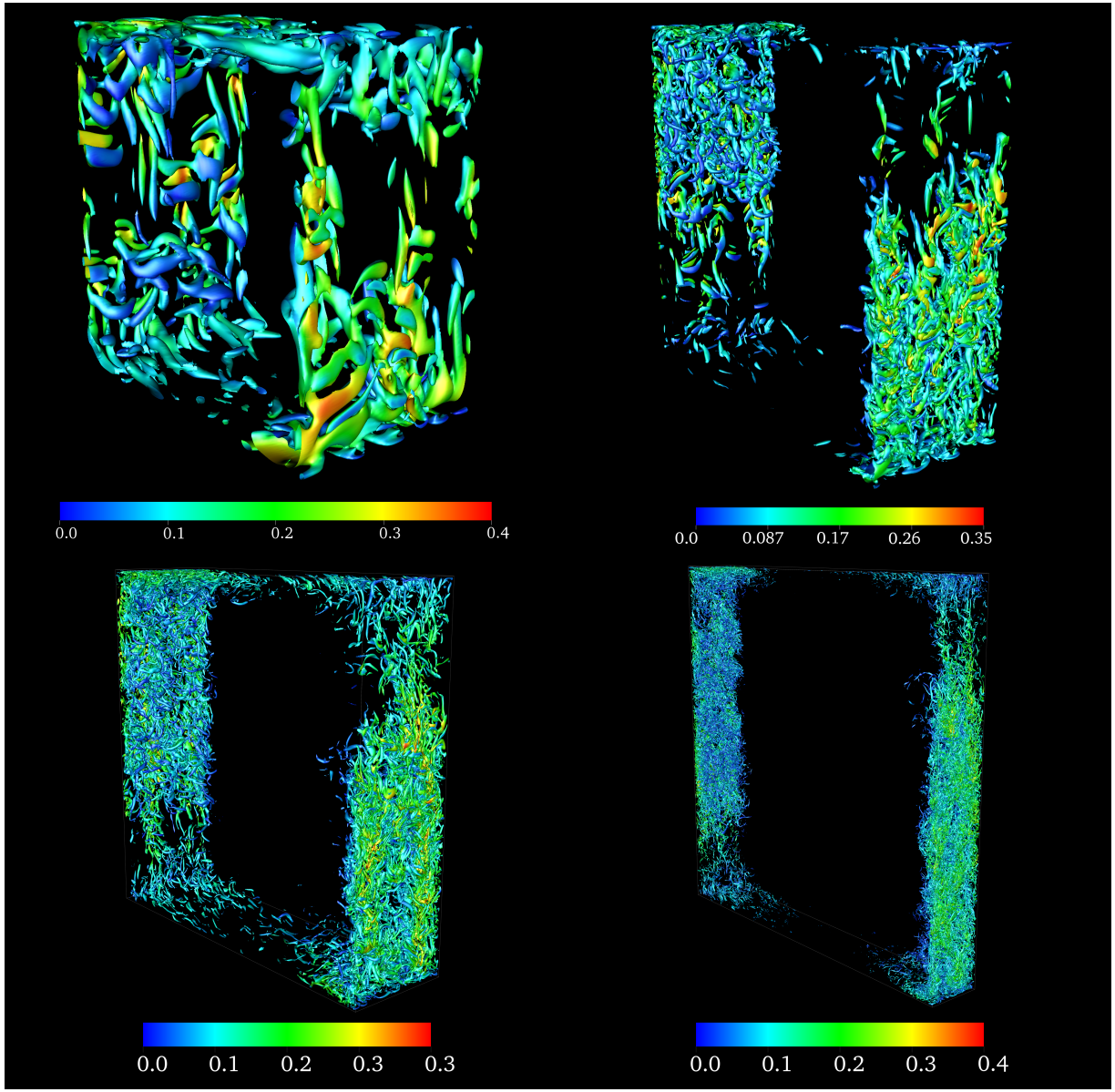
Flow structures: temperature transient (Ra = 1 x 10^{10})



Flow structures: iso Q surfaces

1.0×10^{8}

1.58×10^{9}



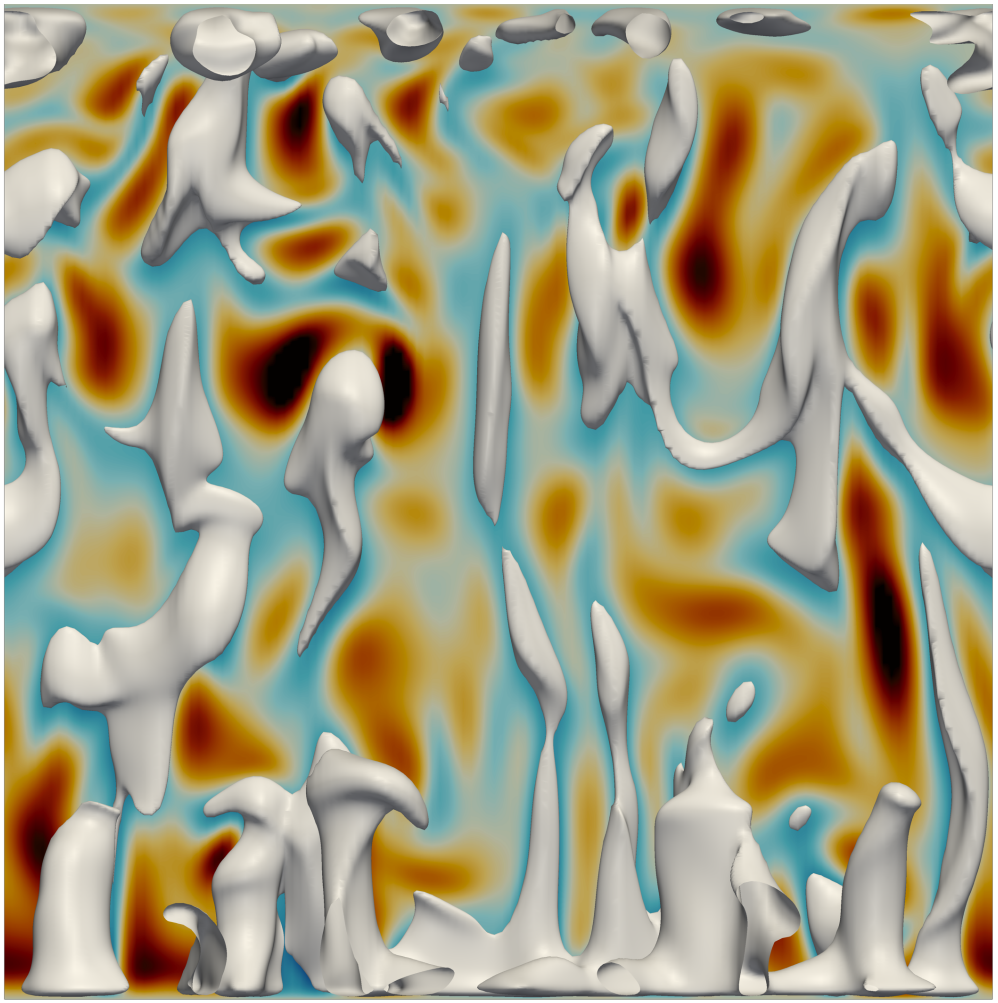
1.0×10^{10}

1.0×10^{11}

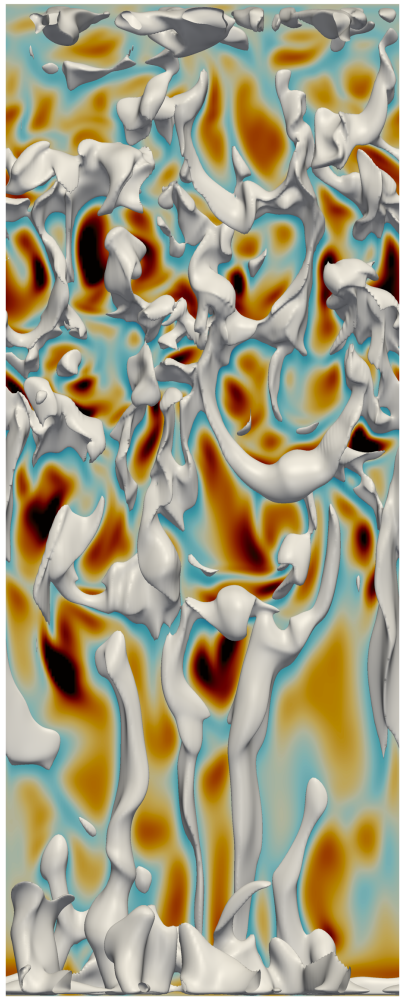
Imperial College London

Flow structures: iso-temperature

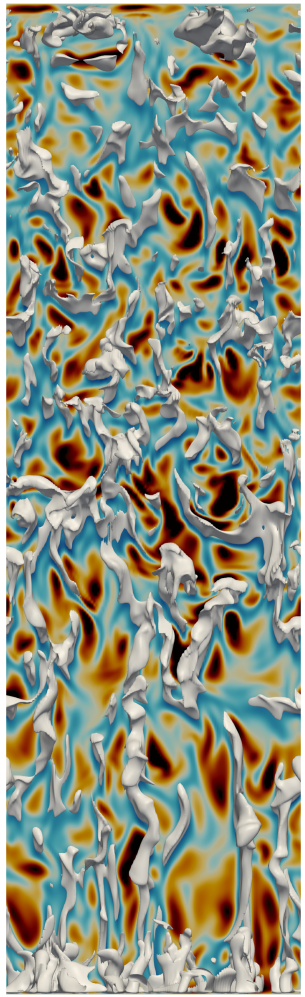
1.0×10^{8}



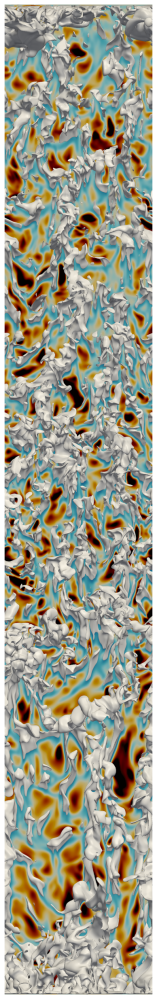
1.58×10^{9}



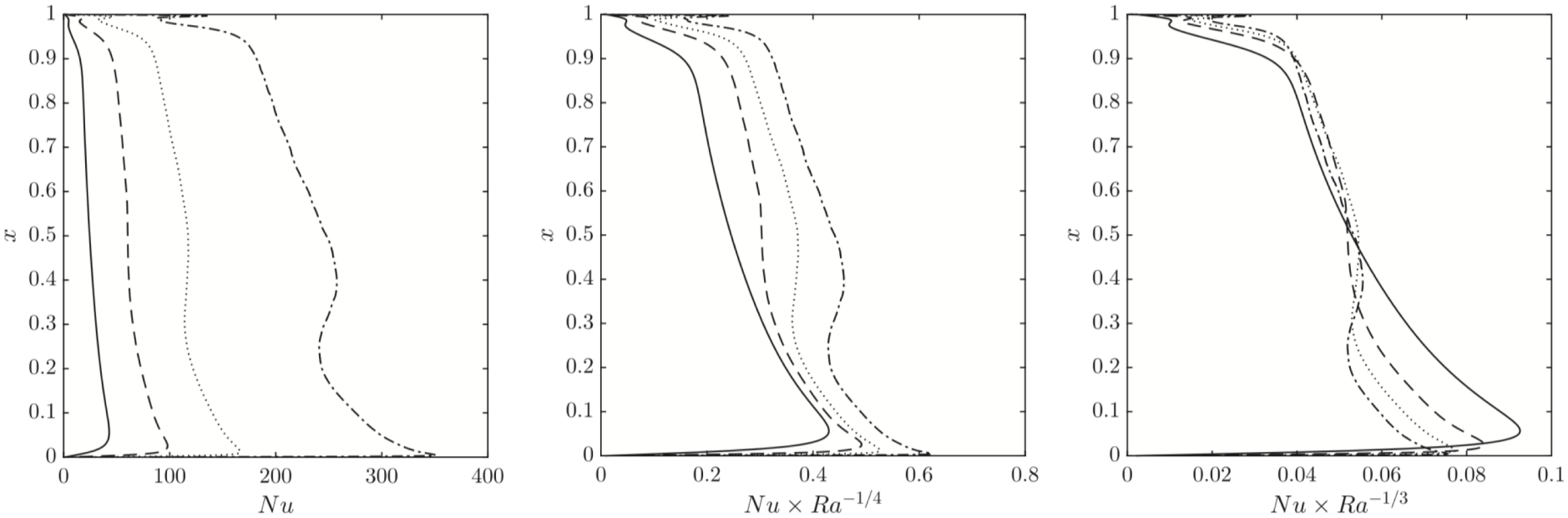
1.0×10^{10}



1.0×10^{11}

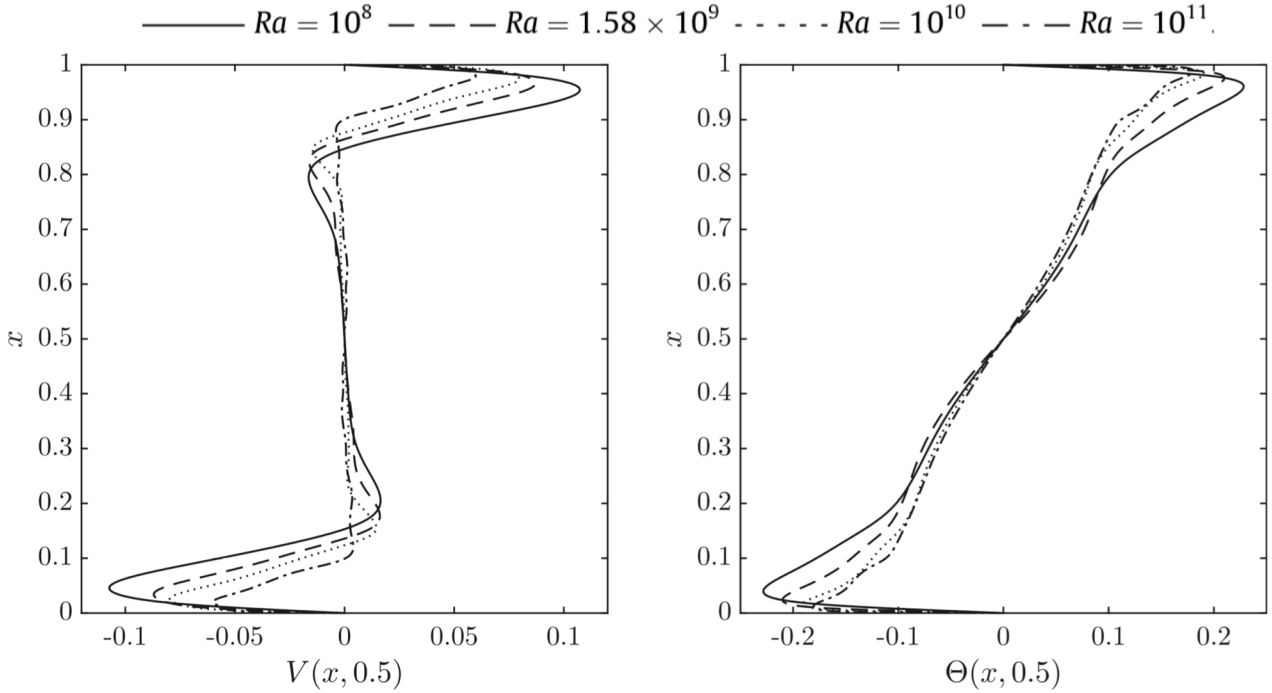


Nusselt on the hot wall

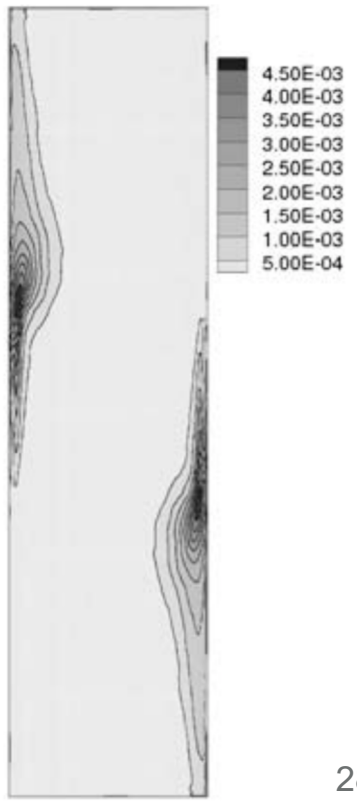


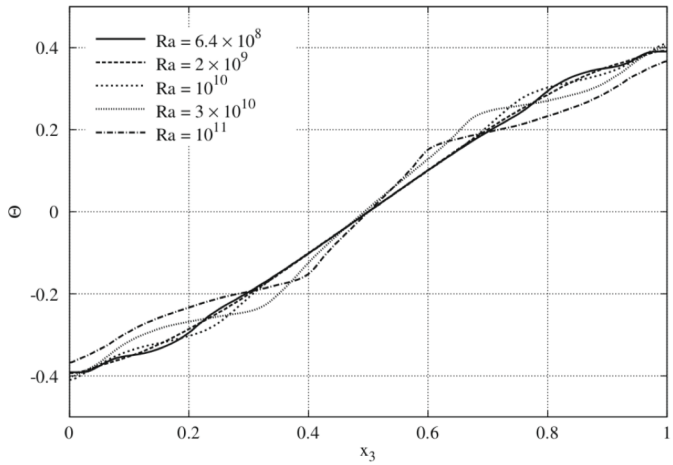
——— $Ra = 10^8$ — — — $Ra = 1.58 \times 10^9$ - - - - - $Ra = 10^{10}$ — - — $Ra = 10^{11}$.

Thermal stratification



Thermal stratification parameter decreases with Ra

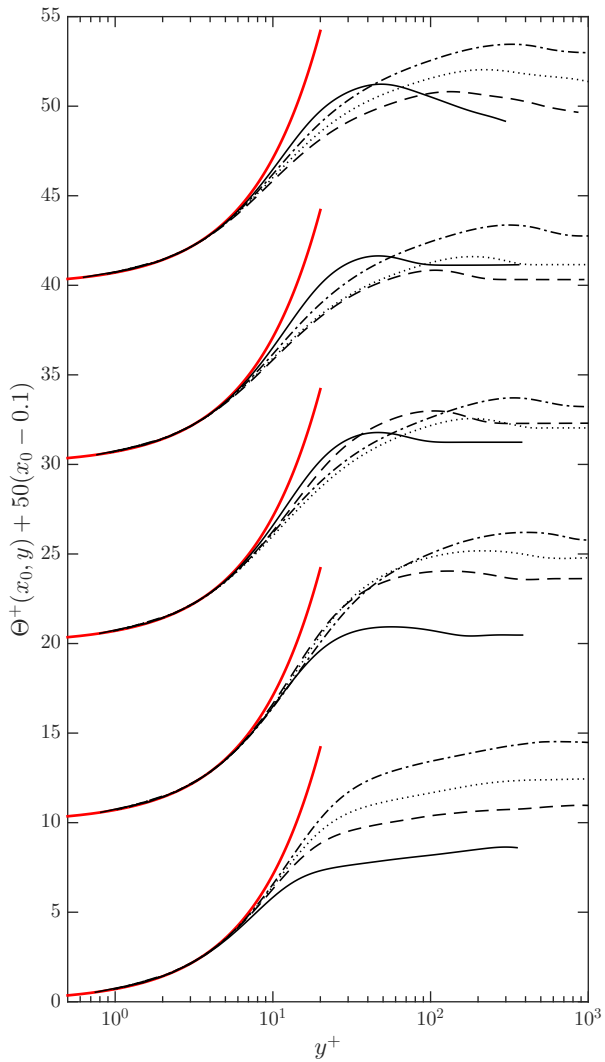


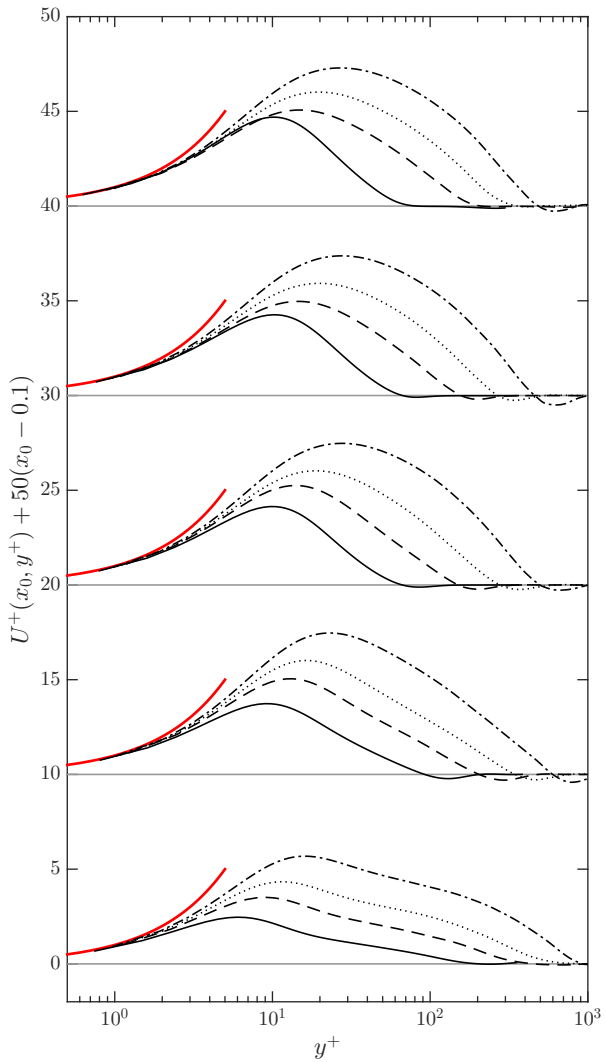


Trias et al. (2010): Thermal stratification parameter increases with Ra

Imperial College London

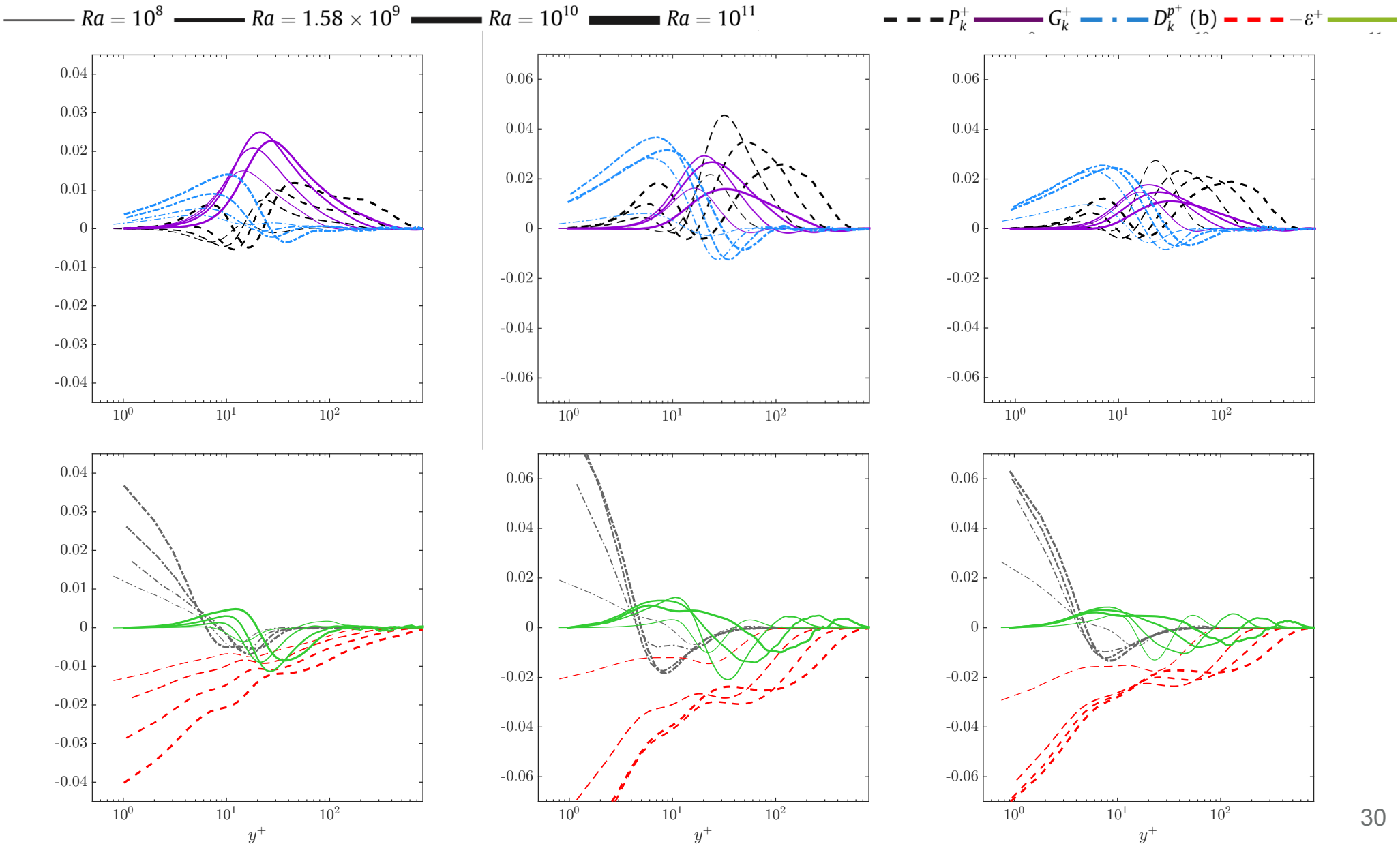
Mean velocity and temperature along hot wall





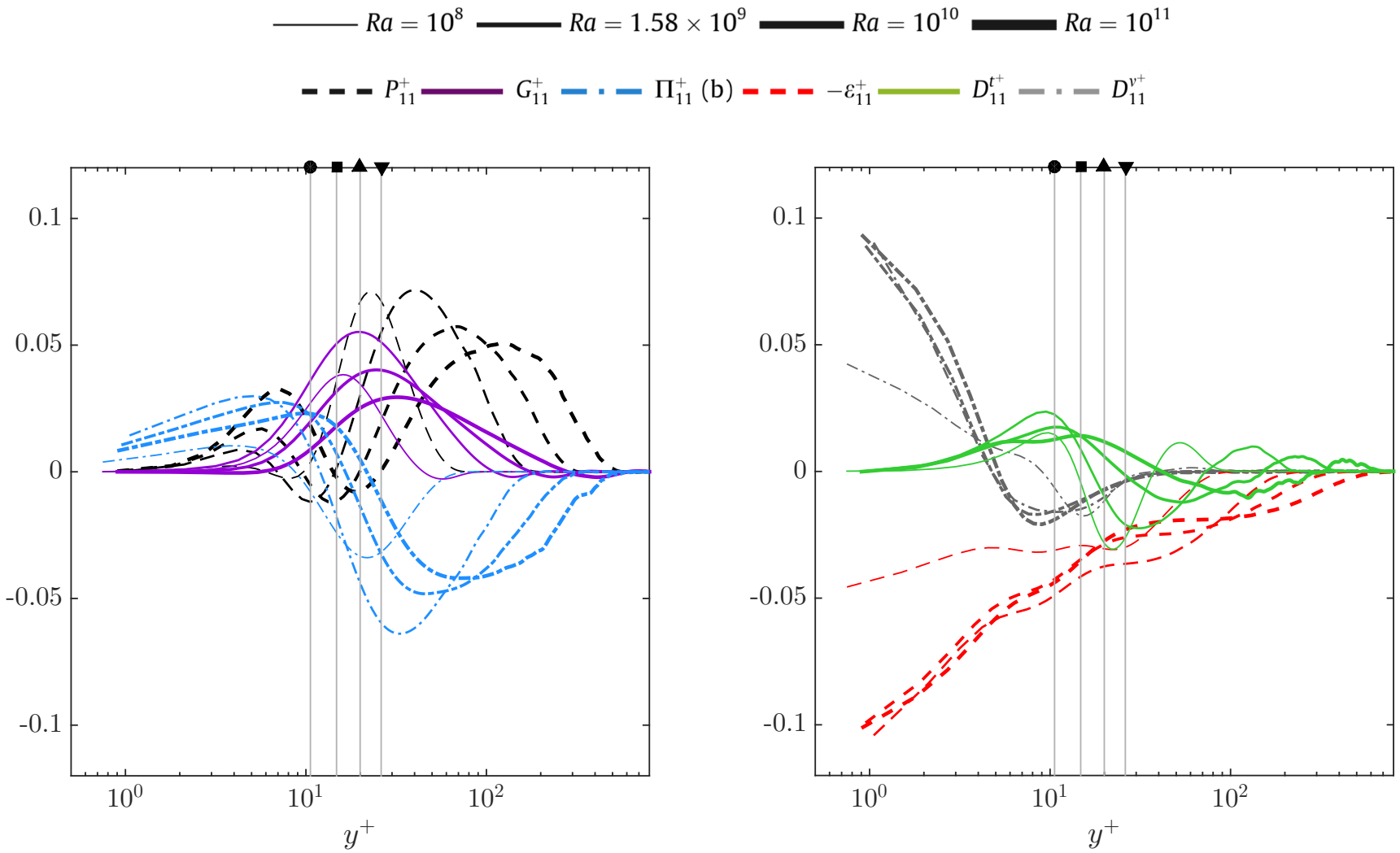
Imperial College London

TKE budget at x/H = 0.2 – 0.5 – 0.8



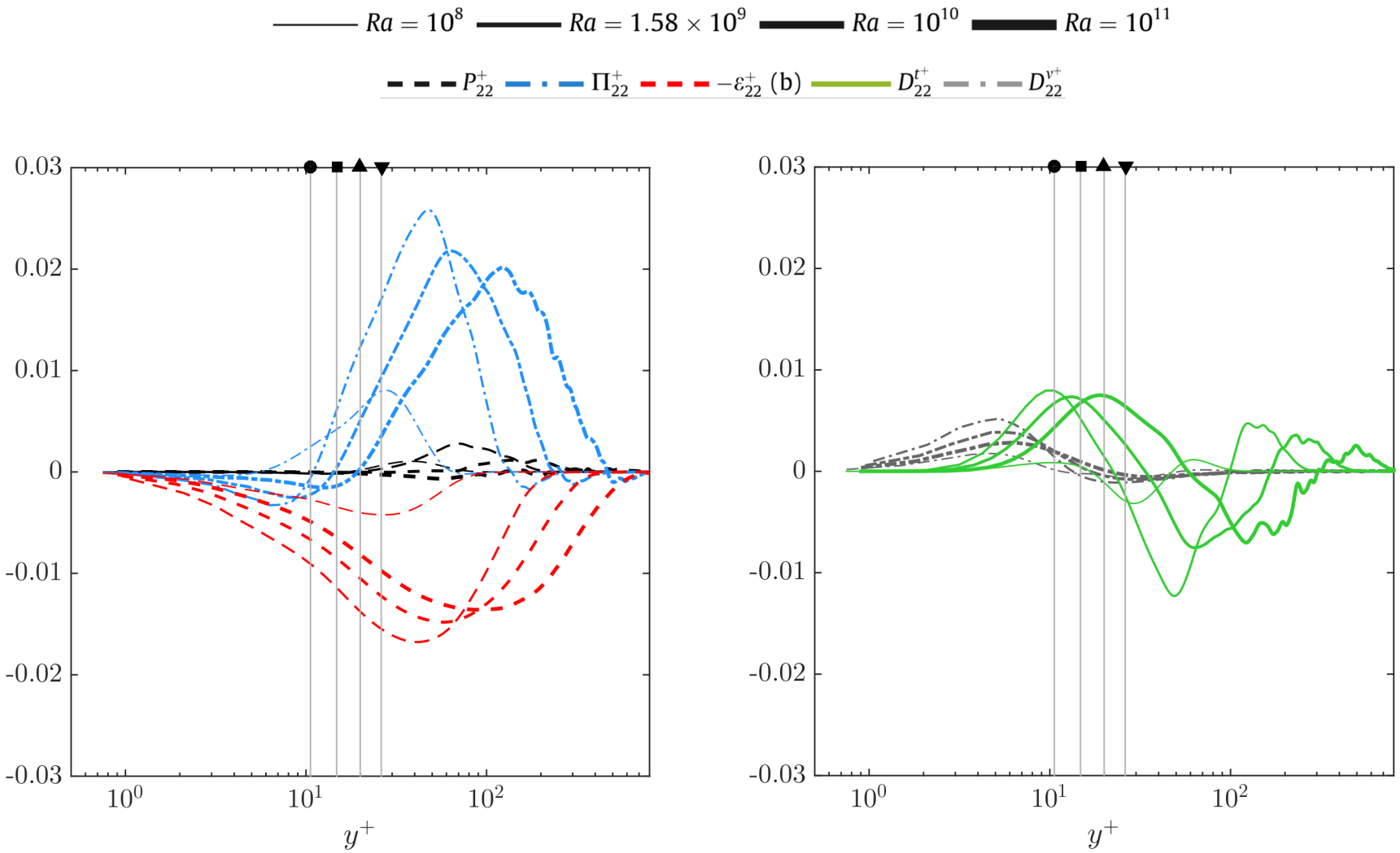
Imperial College London

uu budget at x/H = 0.8



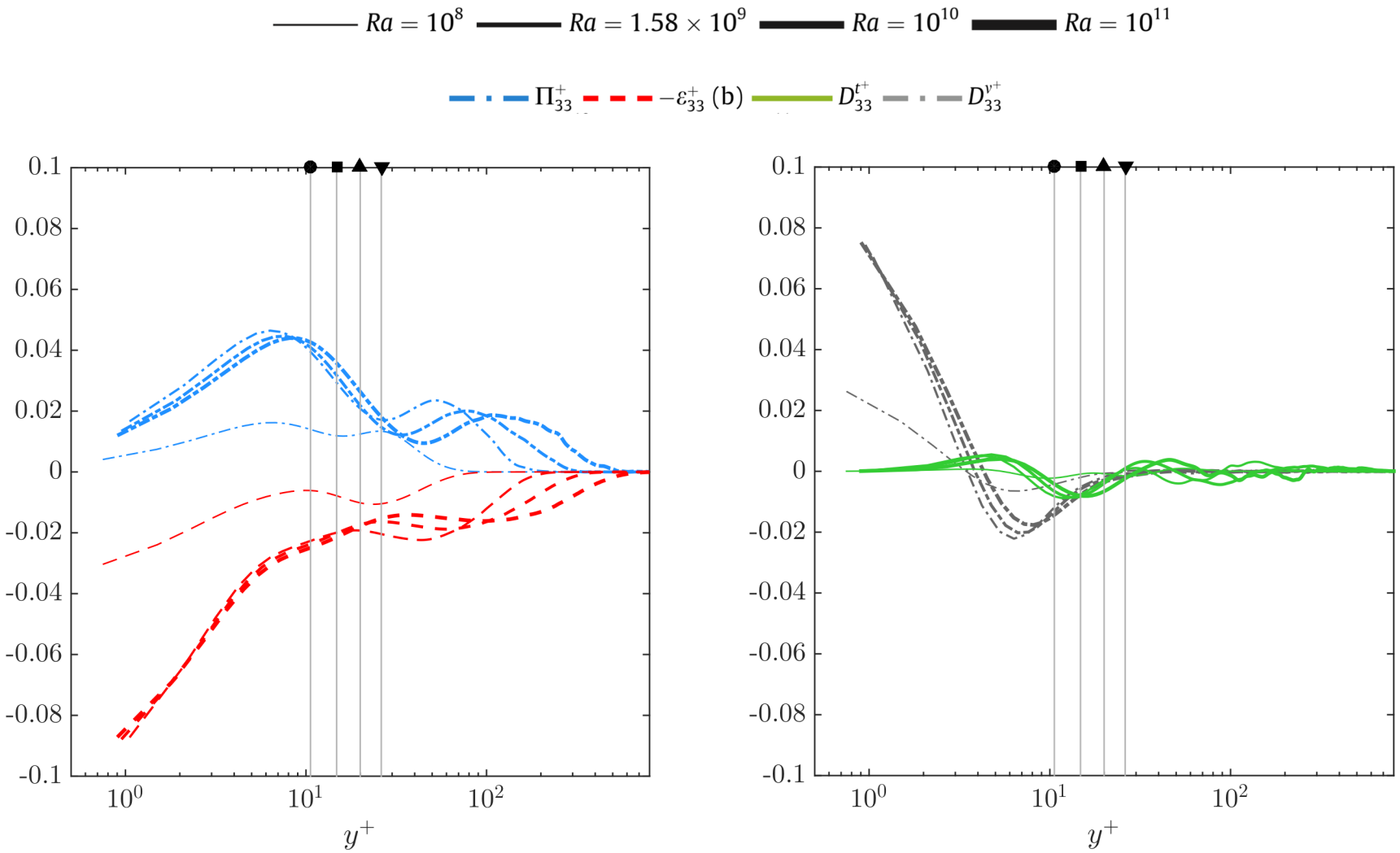
Imperial College London

vv budget at x/H = 0.8

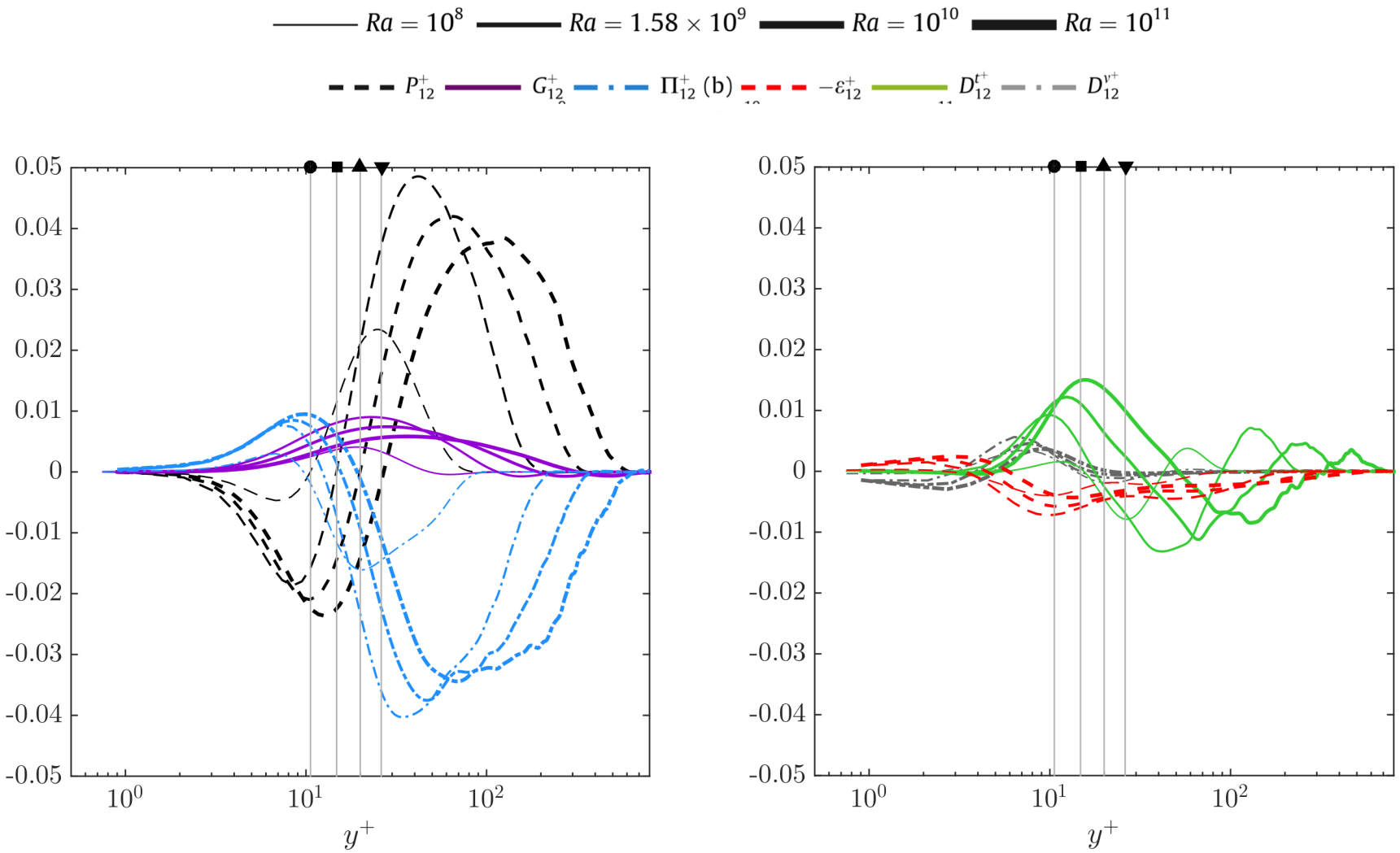


Imperial College London

ww budget at x/H = 0.8

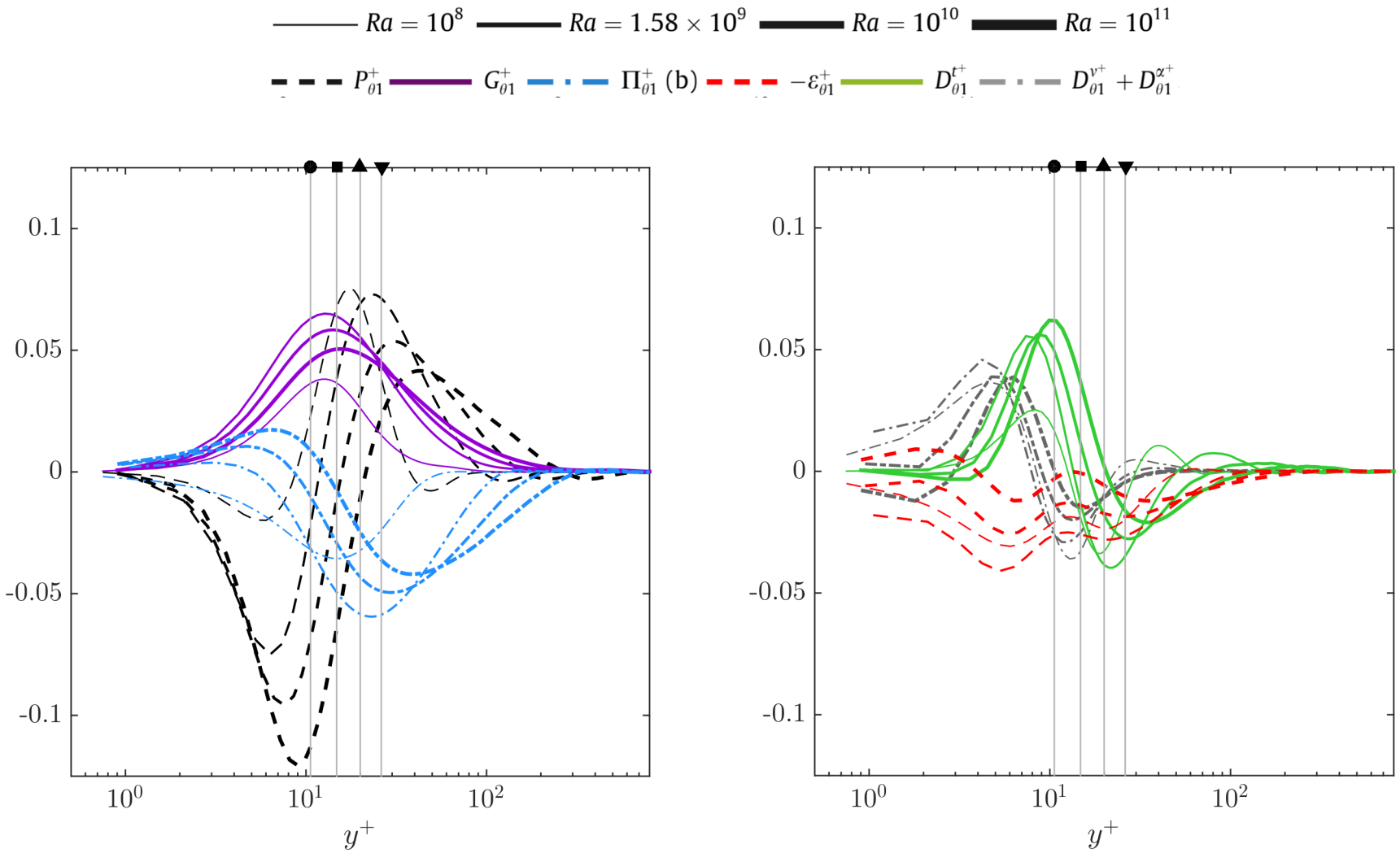


uv budget at x/H = 0.8



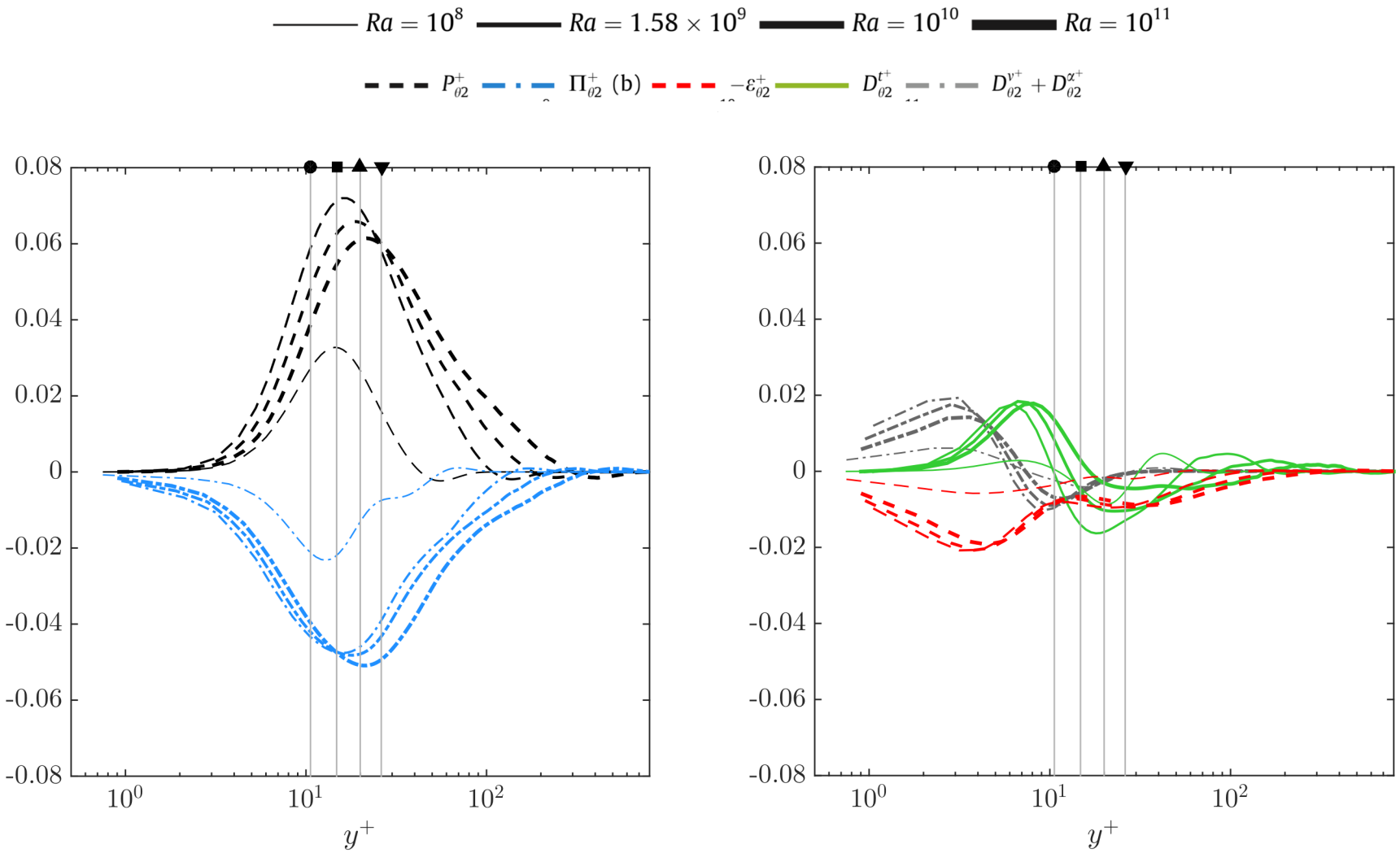
Imperial College London

uT budget at x/H = 0.8



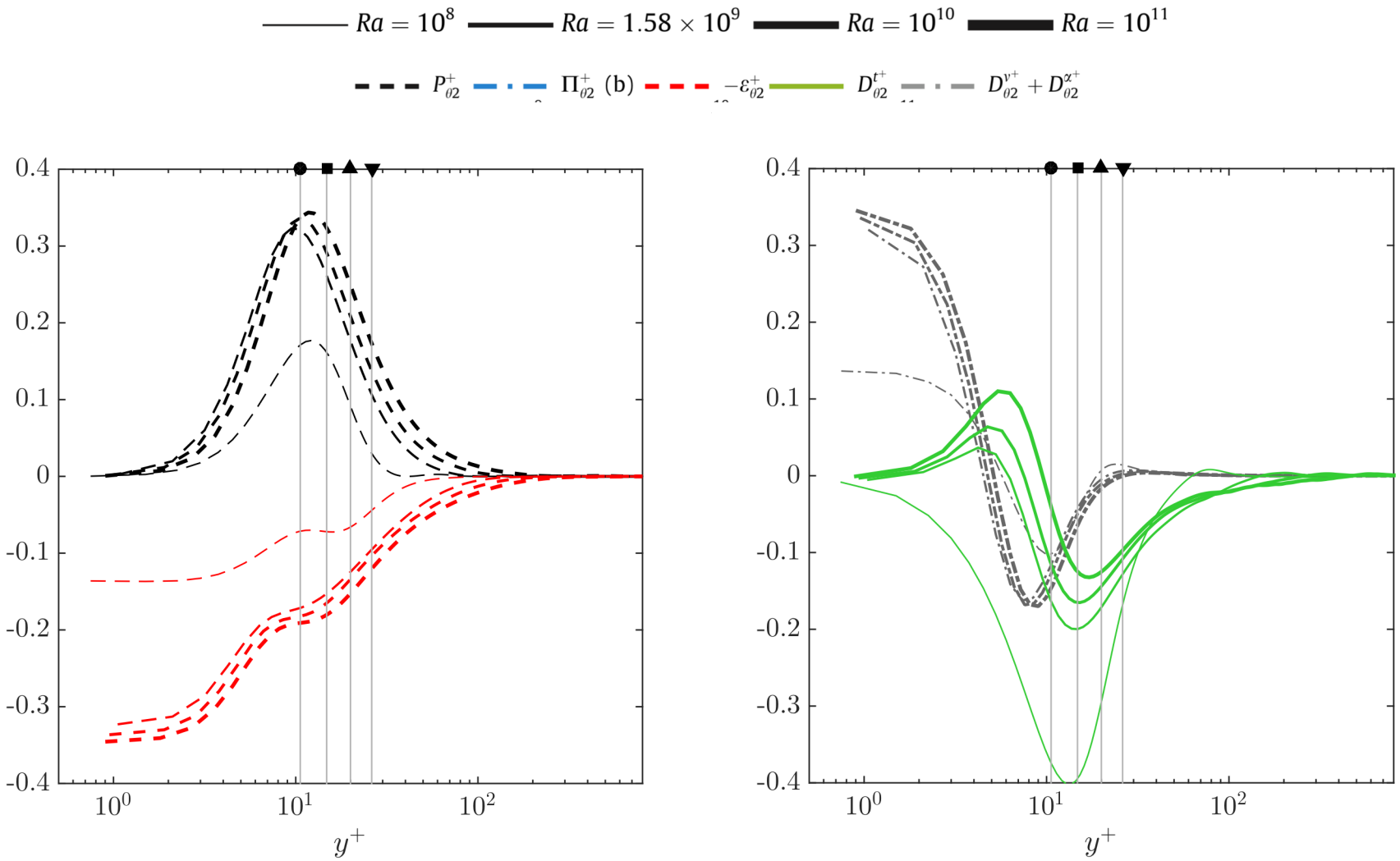
Imperial College London

vT budget at x/H = 0.8

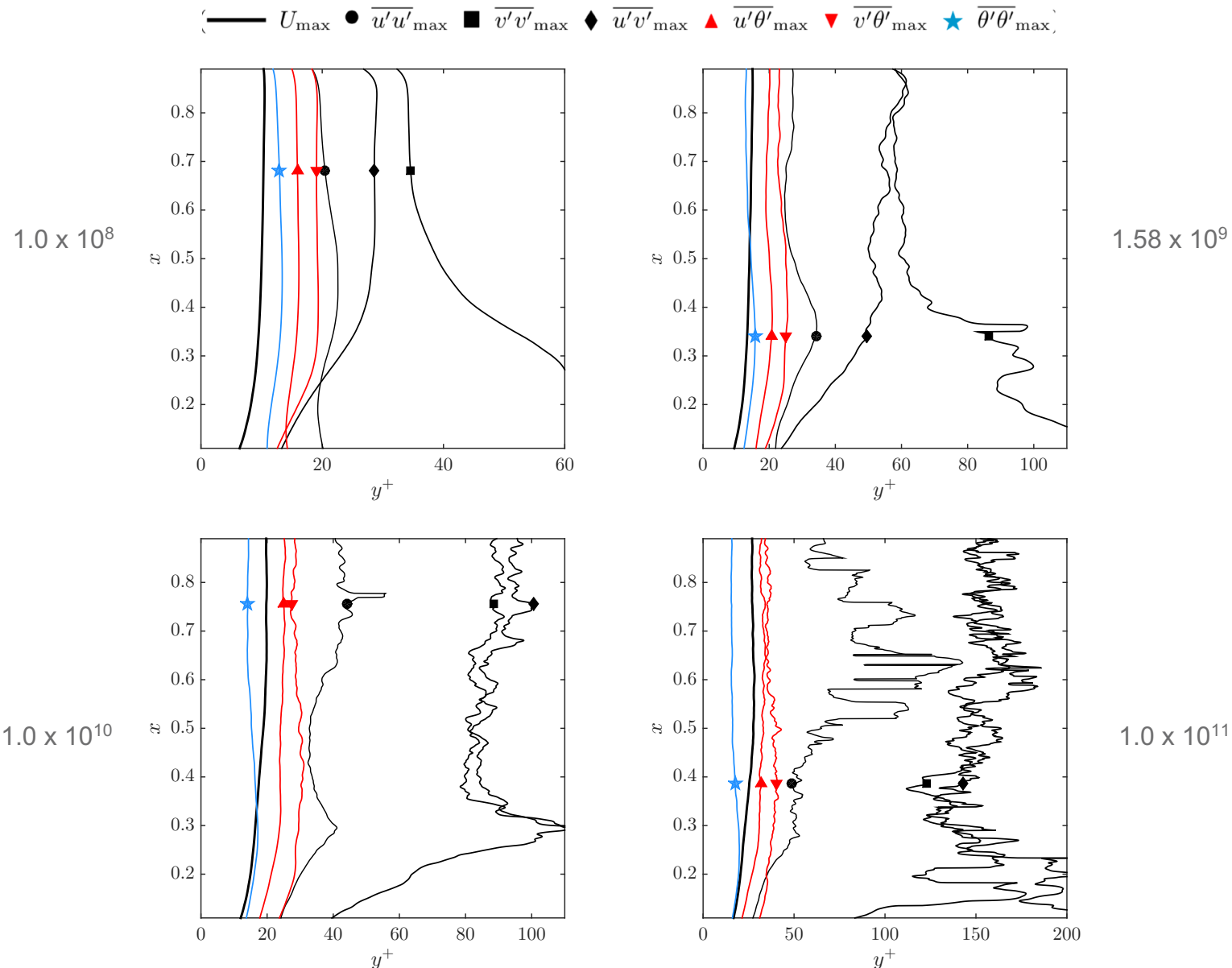


Imperial College London

TT budget at x/H = 0.8



Extremas position in hot BL



EBRSM comparison against DNS

EBRSM formulation (1/2)

- Implementation in OpenFOAM 2.3.0 of the EBRSM version of Dehoux (2012

$$\frac{\partial R_{ij}}{\partial t}+U_k\frac{\partial R_{ij}}{\partial x_k}=P_{ij}+G_{ij}+D_{ij}^{\nu}+D_{ij}^{t}+\phi_{ij}^{\star}-\varepsilon_{ij}$$

$$P_{ij}=-\left(R_{ik}\frac{\partial U_j}{\partial x_k}+R_{jk}\frac{\partial U_i}{\partial x_k}\right) \qquad G_{ij}=-\beta\left(g_i\overline{u_j'\theta'}+g_j\overline{u_i'\theta'}\right) \qquad D_{ij}^{t}=C_s\frac{\partial}{\partial x_k}\left(T_{durb}R_{km}\frac{\partial R_{ij}}{\partial x_m}\right) \qquad D_{ij}^{\nu}=\nu\frac{\partial^2 R_{ij}}{\partial x_k\partial x_k}$$

$$\phi_{ij}^{\star}=\left(1-\alpha^3\right)\phi_{ij}^{w}+\alpha^3\phi_{ij}^{h}$$

$$\phi_{ij}^{w}=\frac{\varepsilon}{\tau_{lim}^{w}}\left(R_{ik}n_jn_k+R_{jk}n_in_k-\frac{1}{2}R_{kl}n_kn_l\left(n_in_j+\delta_{ij}\right)\right) \quad \text{and} \quad \mathbf{n}=\frac{\nabla\alpha}{\left\|\nabla\alpha\right\|}$$

$$\phi_{ij}^{h}=-\left(C_1\varepsilon+C_2^{\star}P_k\right)a_{ij}+\left(C_3-C_3^{\star}\sqrt{a_{kl}a_{kl}}\right)k\bar{S}_{ij}+C_4k\left(a_{ik}S_{jk}+a_{jk}S_{ik}-\frac{2}{3}a_{kl}\bar{S}_{kl}\delta_{ij}\right)+C_5k\left(a_{ik}\Omega_{jk}+a_{jk}\Omega_{ik}\right)-C_6\left(G_{ij}-\frac{2}{3}G_k\delta_{ij}\right)$$

$$\varepsilon_{ij}=\left(1-\alpha^3\right)\varepsilon_{ij}^{w}+\alpha^3\varepsilon_{ij}^{h}$$

$$\varepsilon_{ij}^{w}=\frac{R_{ij}}{\tau_{lim}^{w}} \qquad \varepsilon_{ij}^{h}=\frac{2}{3}\frac{k}{\tau_{lim}^{h}}\delta_{ij}$$

EBRSM formulation (2/2)

$$\frac{\partial \varepsilon_{ij}}{\partial t} + U_k \frac{\partial \varepsilon_{ij}}{\partial x_k} = D_\varepsilon^t + C_{\varepsilon 1}\left(1 + A_1^p\left(1-\alpha^3\right)\frac{P_k}{\varepsilon}\right)\frac{P_k}{T_{durb}} + G_k^\varepsilon - C_{\varepsilon 2}\frac{\varepsilon}{T_{durb}} + \frac{\partial}{\partial x_j}\left(\frac{\nu}{\sigma_\varepsilon^\nu}\frac{\partial \varepsilon}{\partial x_k}\right)$$

$$D_\varepsilon^t = \frac{\partial}{\partial x_k}\left(C_s^\varepsilon T_{durb} R_{km}\frac{\partial \varepsilon}{\partial x_m}\right) \qquad P_k = \frac{1}{2}P_{ii} \qquad G_k = \frac{1}{2}G_{ii}$$

$$\alpha - L^2\nabla^2\alpha = 1$$

$$T_{durb} = \max\left(\frac{k}{\varepsilon}, C_T\sqrt{\frac{\nu}{\varepsilon}}\right) \qquad \tau_{lim}^w = \max\left(\frac{k}{\varepsilon}, 10\alpha^3\sqrt{\frac{\nu}{\varepsilon}}\right) \qquad \tau_{lim}^h = \max\left(\frac{k}{\varepsilon}, 10\sqrt{\frac{\nu}{\varepsilon}}\right) \qquad L = C_L\max\left(C_\eta\frac{\nu^{3/4}}{\varepsilon^{1/4}}, \frac{k^{3/2}}{\varepsilon}\right)$$

C_1	$C_2^\star$	C_3	$C_3^\star$	C_4	C_5	C_6	σ_k	σ_ε^ν	σ_ε
1.7	0.9	0.8	0.65	0.625	0.2	0.5	1.0	1.1	1.15
C_S^ε	C_{ε_1}	C_{ε_2}	C_{ε_3}	A_1^P	C_s	C_L	C_η	C_μ	C_T
0.183	1.44	1.83	2.02	0.1	0.21	0.133	80	0.07	6.0

EBDFM formulation (1/2)

- Implementation in OpenFOAM 2.3.0 of the EBDFM version of Dehoux (2012):

$$\frac{\partial \overline{u_i'\theta'}}{\partial t} + U_j \frac{\partial \overline{u_i'\theta'}}{\partial x_j} = P_{\theta i}^U + P_{\theta i}^T + G_{\theta i} + \phi_{\theta i}^\star - \varepsilon_{\theta i} + D_{\theta i}^t + D_{\theta i}^\nu$$

$$P_{\theta i}^U = -R_{ij}\frac{\partial \Theta}{\partial x_j} \qquad P_{\theta i}^T = -\overline{u_i'\theta'}\frac{\partial U_i}{\partial x_j} \qquad G_{\theta i} = -\beta g_i \overline{\theta'\theta'} \qquad D_{\theta i}^t = \frac{\partial}{\partial x_k}\left(C_\theta^t R_{kl}\tau \frac{\partial \overline{u_i'\theta'}}{\partial x_l}\right) \qquad D_{\theta i}^\nu = \frac{\partial}{\partial x_k}\left(\frac{\kappa+\nu}{2}\frac{\partial \overline{u_i'\theta'}}{\partial x_k}\right)$$

$$\phi_{\theta i}^\star - \varepsilon_{\theta i} = \left(1-\alpha_\theta\right)\left(\phi_{\theta i}^w - \varepsilon_{\theta i}^w\right) + \alpha_\theta\left(\phi_{\theta i}^h - \varepsilon_{\theta i}^h\right)$$

$$\phi_{\theta i}^h = -\frac{\sqrt{Pr}}{\sqrt{R\tau}} C_{\theta 1}\overline{u_i'\theta'} - C_{\theta 2}P_{\theta i}^U - C_{\theta 3}G_{\theta i} \qquad \varepsilon_{\theta i}^h = 0$$

$$\phi_{\theta i}^h = -\frac{\sqrt{Pr}}{\sqrt{R\tau}}\left[1 + C_{w\theta}^\phi\left(1-\alpha_\theta\right)\frac{P_k+G_k}{\varepsilon}\right]\overline{u_i'\theta'}n_i n_j \qquad \varepsilon_{\theta i}^w = \frac{\sqrt{Pr}}{\sqrt{R\tau}}C_\varepsilon\left[1 + C_{w\theta}^\varepsilon\left(1-\alpha_\theta\right)\frac{P_k+G_k}{\varepsilon}\right]\left(\overline{u_i'\theta'} + \overline{u_j'\theta'}n_i n_j\right)$$

$$\alpha_\theta - L_\theta^2\nabla^2\alpha_\theta = 1 \quad \text{where} \quad L_\theta = 2.5L$$

$C_{\theta 1}$	$C_{\theta 2}$	$C_{\theta 3}$	C_ε	$C_{w\theta}^\phi$	$C_{w\theta}^\varepsilon$	τ
4.15	0.3	0.5	$\frac{1}{2}\left(1+\frac{1}{\mathrm{Pr}}\right)$	2.0	−0.3	k/ε

EBDFM formulation (2/2)

$$\frac{\partial \overline{\theta'\theta'}}{\partial t} + U_j \frac{\partial \overline{\theta'\theta'}}{\partial x_j} = P_{\theta\theta} - \varepsilon_{\theta\theta} + D^t_{\theta\theta} + \frac{\partial}{\partial x_k}\left(\frac{\nu}{\mathrm{Pr}} \frac{\partial \overline{\theta'\theta'}}{\partial x_k} \right)$$

$$P_{\theta\theta} = -2\overline{u_i'\theta'} \frac{\partial \Theta}{\partial x_i}$$

$$D^t_{\theta\theta} = \frac{\partial}{\partial x_k}\left(C_{\theta\theta} R_{kl} \frac{k}{\varepsilon} \frac{\partial \overline{\theta'\theta'}}{\partial x_l} \right)$$

$$\varepsilon_{\theta\theta} = \frac{\overline{\theta'\theta'}}{R} \frac{\varepsilon}{k}$$

$$R = \left(1 - \alpha_\theta\right)\mathrm{Pr} + \alpha_\theta R^h \quad \text{where} \quad R^h = 0.5$$

$C_{\theta 1}$	$C_{\theta\theta}$	ξ	η	$\sigma_{\theta\theta}$
4.15	0.22	0.7	0.5	1.0

AFM and EB-AFM formulations

- AFM formulation

$$\overline{u_i'\theta'} = -C_\theta \tau \left(R_{ij} \frac{\partial \Theta}{\partial x_i} + \xi \overline{u_j'\theta'} \frac{\partial U_i}{\partial x_j} + \eta \beta g_i \overline{\theta'\theta'} \right)$$

$C_{\theta 1}$	$C_{\theta\theta}$	ξ	η
4.15	0.22	0.7	0.5

- EBAFM formulation

$$\overline{u_i'\theta'} = -C_\theta \frac{k}{\varepsilon} \left(R_{ij} \frac{\partial \Theta}{\partial x_i} + \xi \overline{u_j'\theta'} \frac{\partial U_i}{\partial x_j} + \eta \beta g_i \overline{\theta'\theta'} + \gamma \overline{u_j'\theta'} n_i n_j \right)$$

$$C_\theta = \frac{\sqrt{R} C_\theta'}{\alpha_\theta C_{1\theta} \sqrt{R^h} + \left(1 - \alpha_\theta\right) \sqrt{\mathrm{Pr}} C_\varepsilon}$$

$\mathcal{R}^h$	ξ	η	γ	C_ε	$C_{\theta 1}$	$C_{\theta 2}$	$C_{\theta 3}$
0.5	$1 - \alpha_\theta C_{2\theta}$	$1 - \alpha_\theta C_{3\theta}$	$(1-\alpha_\theta)(1+C_\varepsilon)\frac{\sqrt{\mathrm{Pr}}}{\sqrt{\mathcal{R}}}$	$\frac{1}{2}\left(1+\frac{1}{\mathrm{Pr}}\right)$	4.15	0.3	0.5

Imperial College London

Simulations run

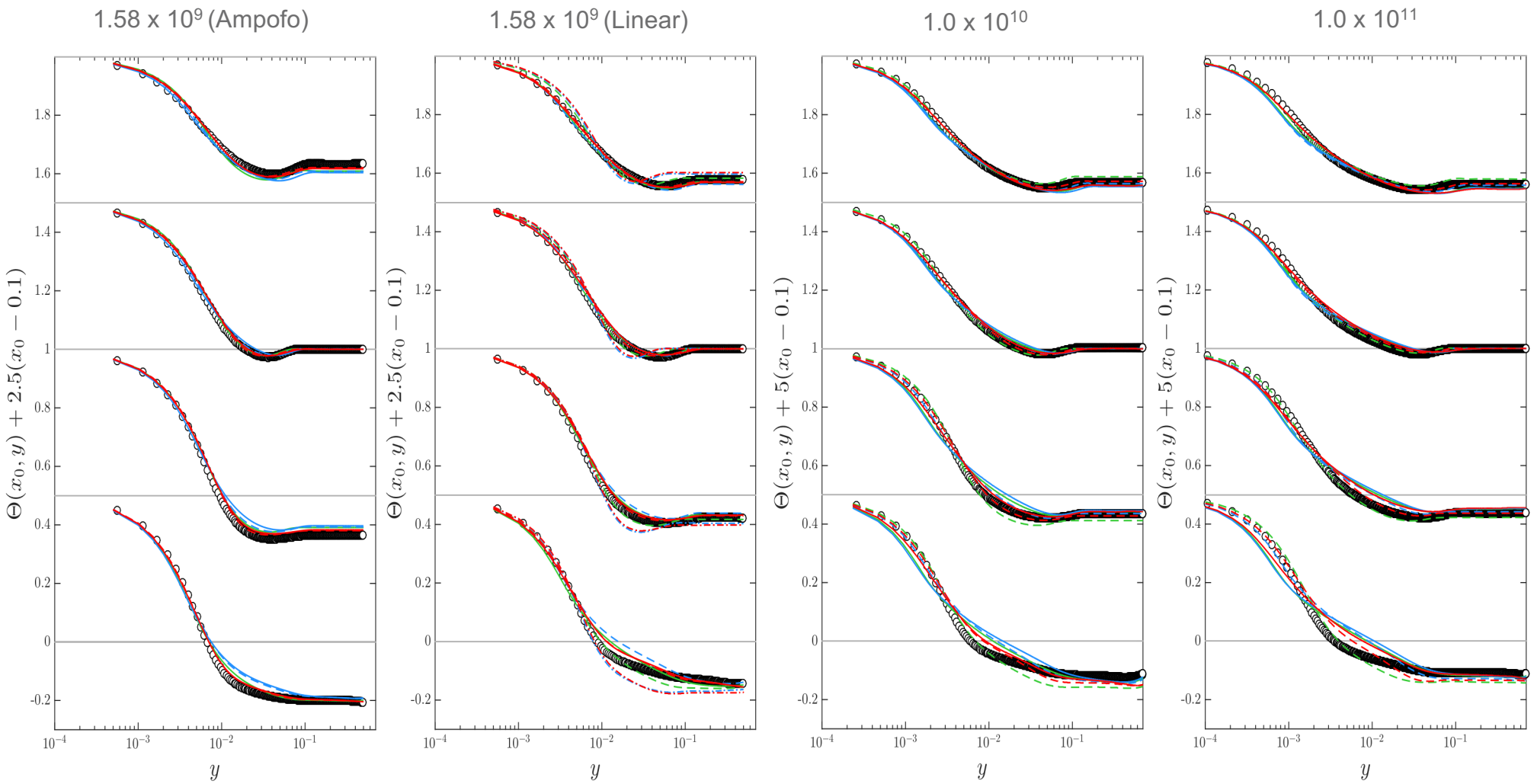
Ra	G_k^{ε}	AFM	EBAFM	EBDFM
1.58×10^9 (Ampofo)	1	S	S	S
	2	Uc	S	S
	3	Uc	Uc	Uc
1.58×10^9	1	Uo	S	Uo
	2	S	S	S
	3	Uc	S	S
1.0×10^{10}	1	Uo	Uo	Uo
	2	Uo	Uo	Uo
	3	Uc	Uc	Uc
1.0×10^{11}	1	Uo	Uo	Uo
	2	Uc	Uo	Uo
	3	Uc	Uc	Uc

S: steady state reached
Uo : Unsteady organised solution
Uc: Unsteady chaotic solution (relaminarised)

$$G_{k1}^{\varepsilon} = C'_{\varepsilon 1} \frac{G_k}{T_{turb}} \qquad G_{k2}^{\varepsilon} = C_{\varepsilon 3} \frac{\sqrt{Pr}}{\sqrt{R}} \frac{G_k}{T_{turb}} \quad with\ C_{\varepsilon 3} = 1.44 \qquad G_{k3}^{\varepsilon} = C_{\varepsilon 3} \frac{\sqrt{Pr}}{\sqrt{R}} \frac{G_k}{T_{turb}} \quad with\ C_{\varepsilon 3} = 2.02$$

Imperial College London

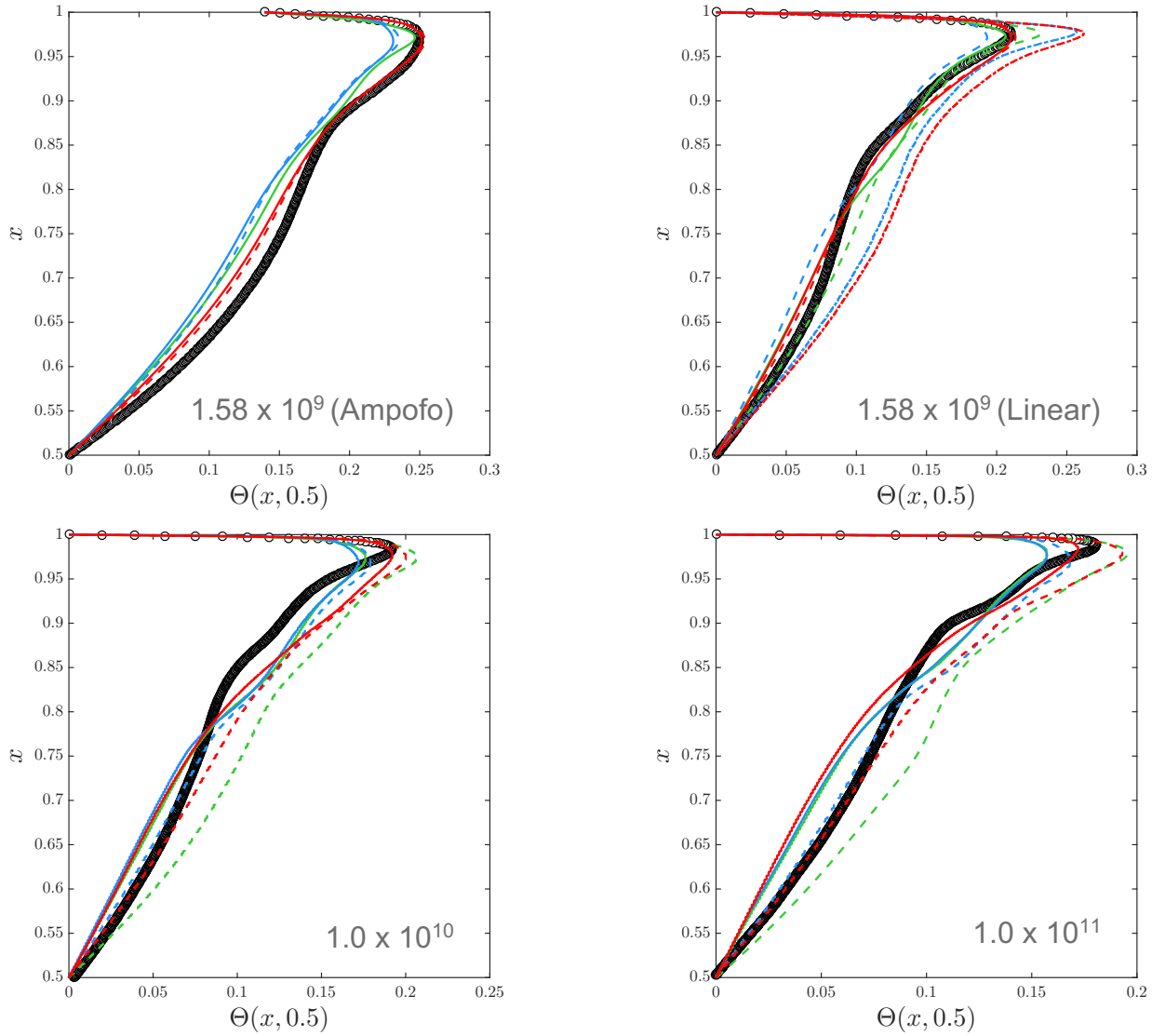
Comparison of mean temperature



○ DNS — AFM — EBAFM — EBDFM

— G^{ε}_{k1} - - - G^{ε}_{k2} -·- G^{ε}_{k3}

Comparison of mean stratification

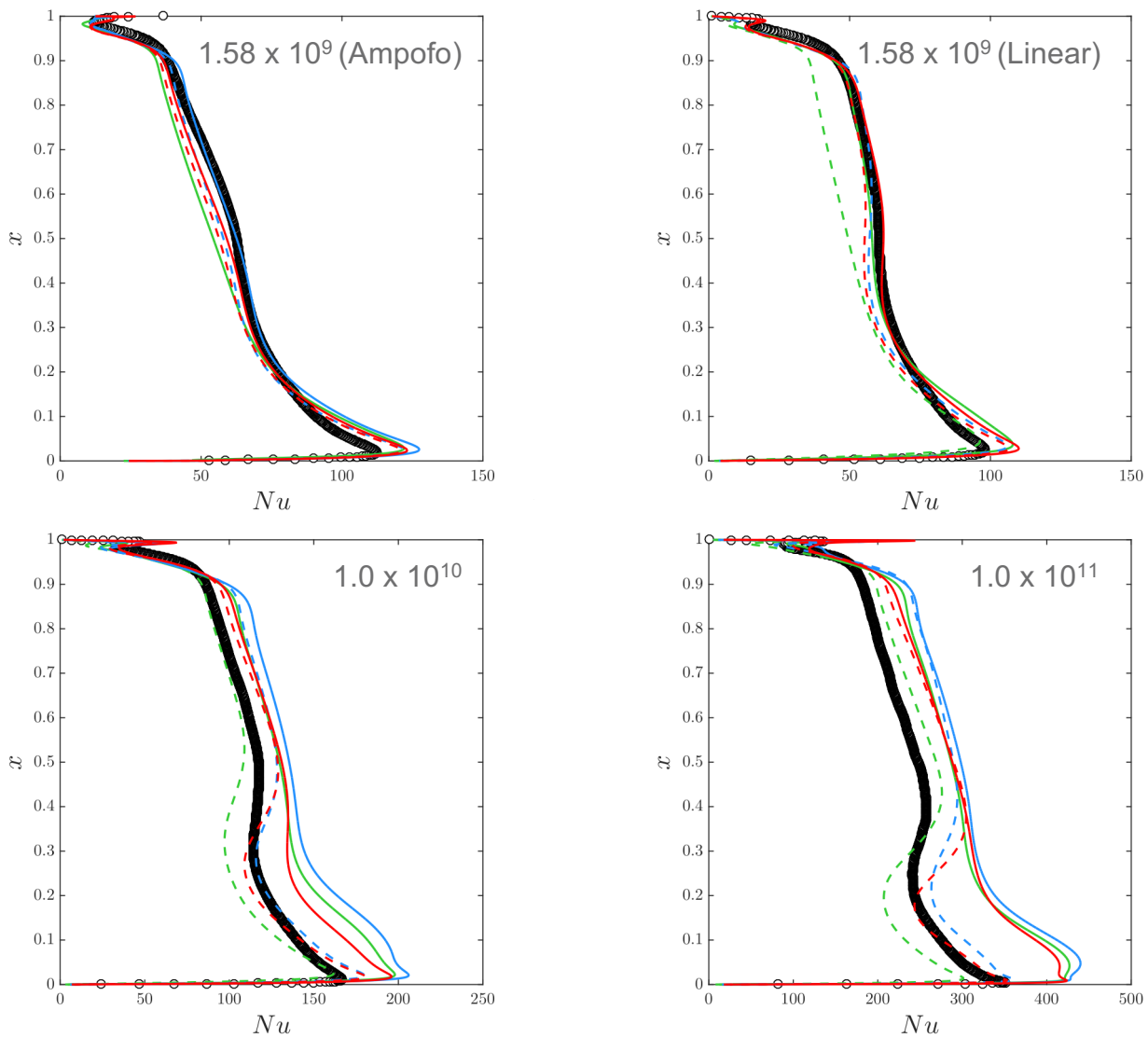


○ DNS — AFM — EBAFM — EBDFM

— G^{ε}_{k1} - - - G^{ε}_{k2} —·— G^{ε}_{k3}

Imperial College London

Comparison of Nusselt (Hot wall)

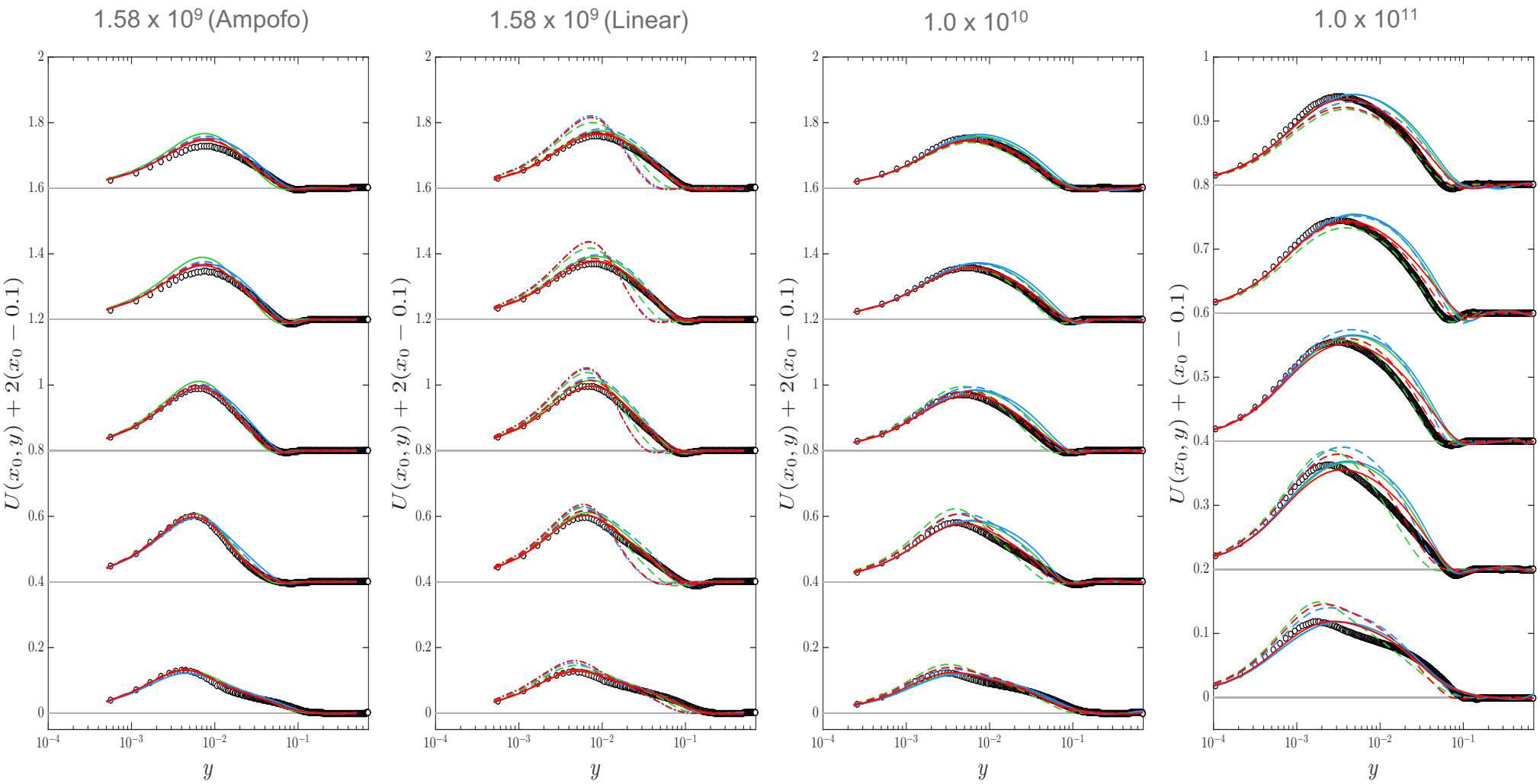


○ DNS — AFM — EBAFM — EBDFM

— G^{ε}_{k1} - - - G^{ε}_{k2} -·- G^{ε}_{k3}

Imperial College London

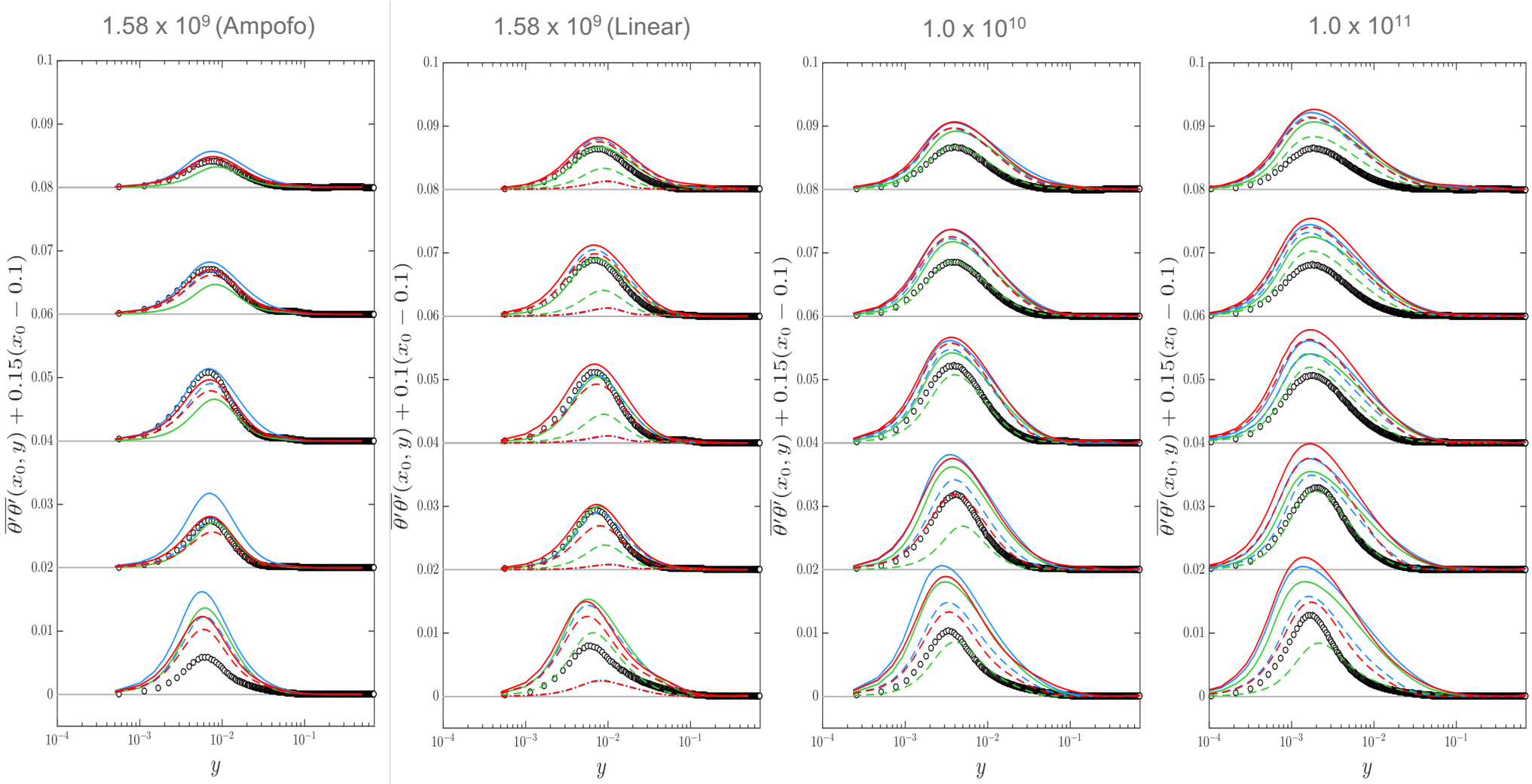
Comparison of Mean velocity U



○ DNS — AFM — EBAFM — EBDFM

— G^{ε}_{k1} - - - G^{ε}_{k2} -·- G^{ε}_{k3}

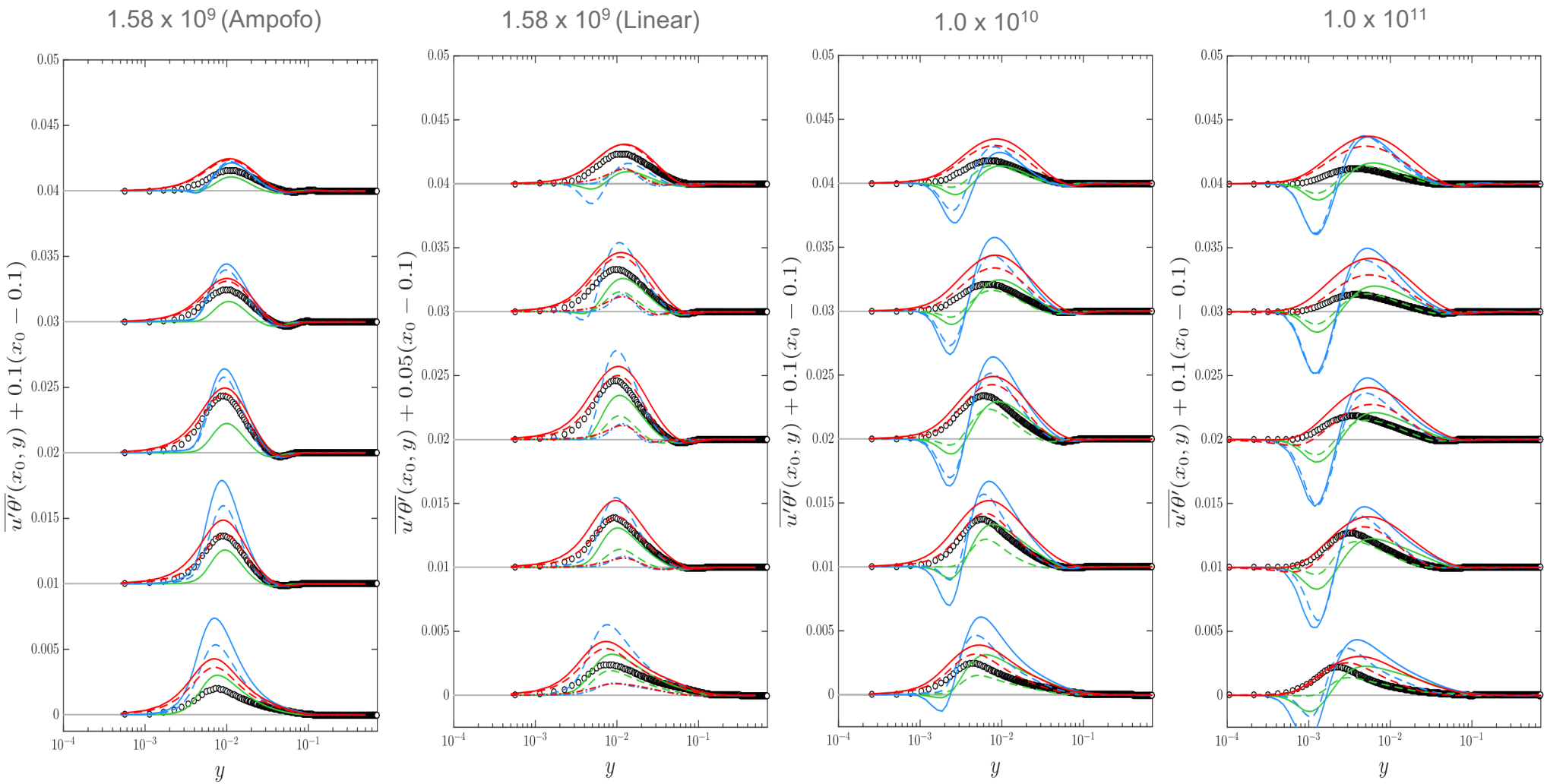
Comparison of temperature variance



○ DNS — AFM — EBAFM — EBDFM — G^{ε}_{k1} - - - G^{ε}_{k2} -·- G^{ε}_{k3}

Imperial College London

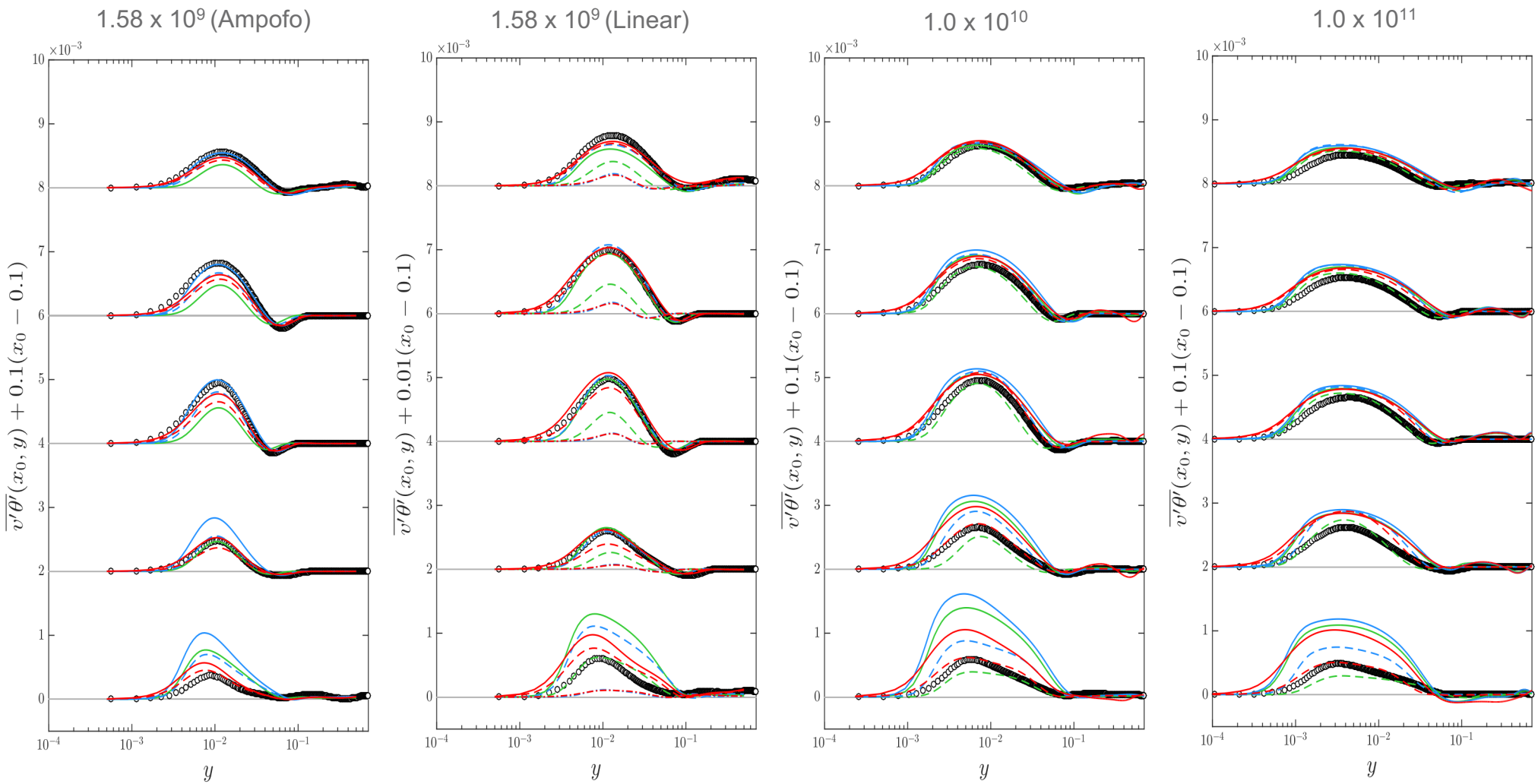
Comparison of turbulent heat flux uT



○ DNS — AFM — EBAFM — EBDFM — G^{ϵ}_{k1} - - - G^{ϵ}_{k2} —·— G^{ϵ}_{k3}

Imperial College London

Comparison of turbulent heat flux vT

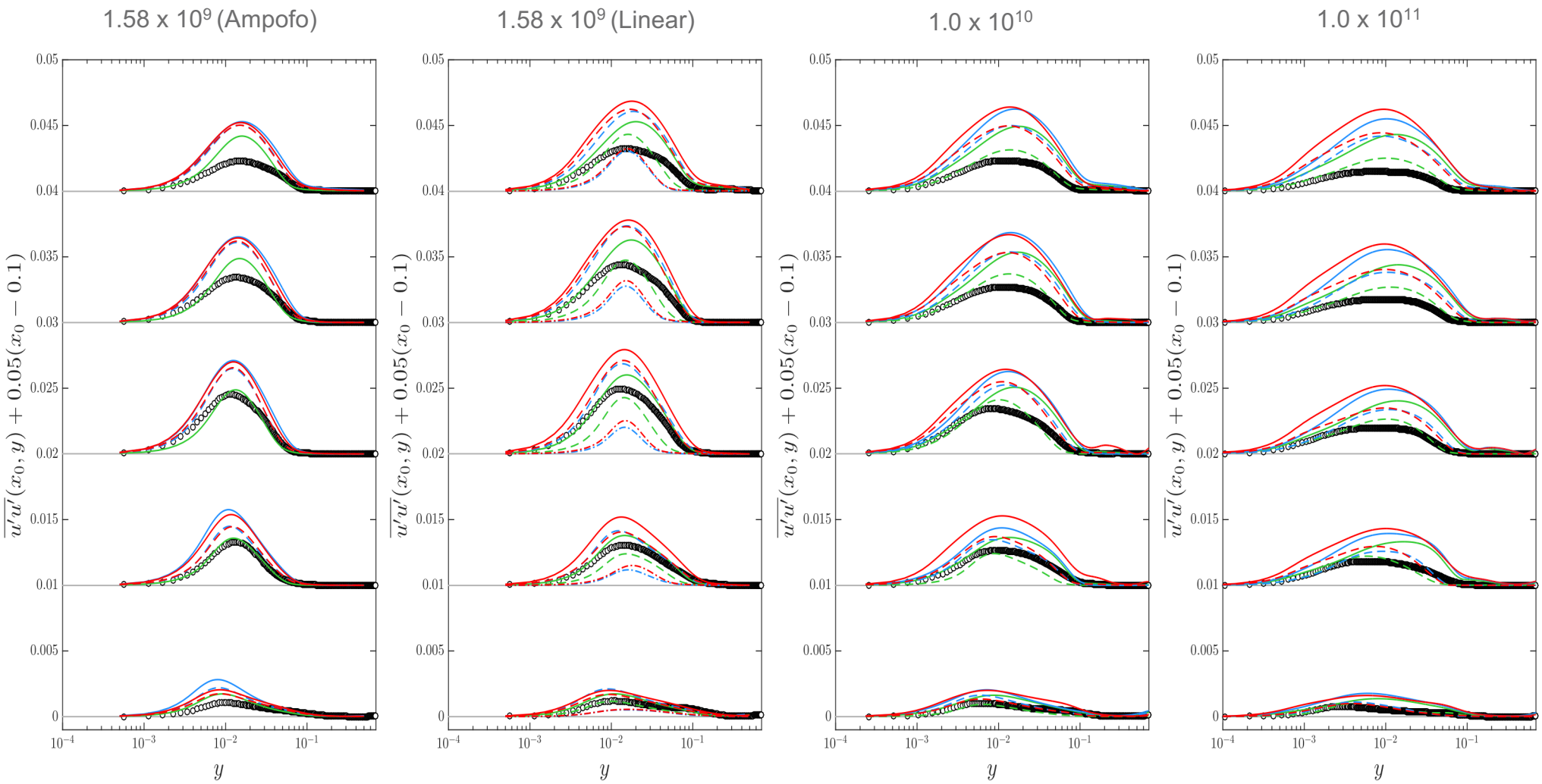


○ DNS — AFM — EBAFM — EBDFM

— G^{ε}_{k1} - - - G^{ε}_{k2} —·— G^{ε}_{k3}

Imperial College London

Comparison of Reynolds stresses: uu

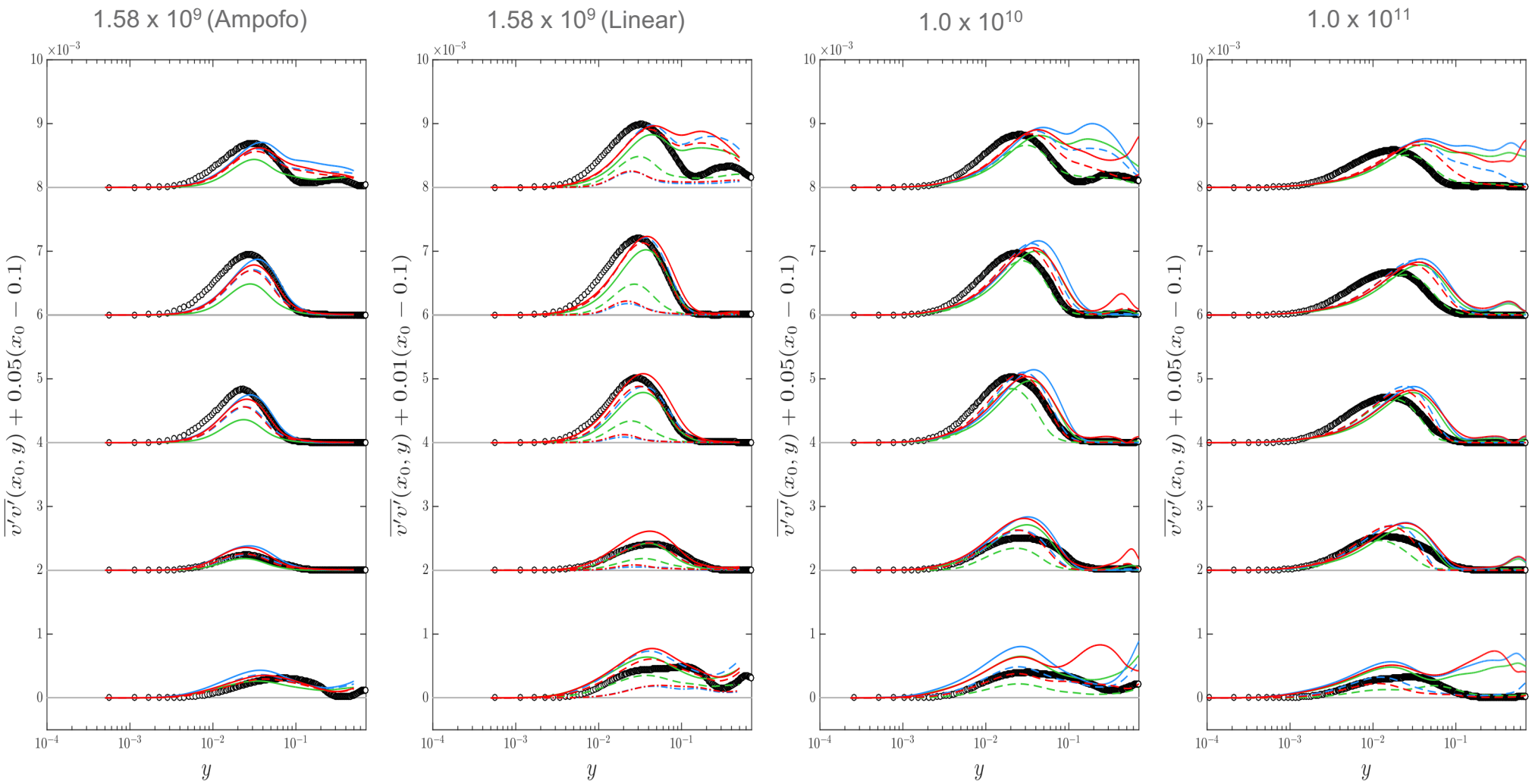


○ DNS — AFM — EBAFM — EBDFM

— G^{ε}_{k1} - - - G^{ε}_{k2} -·- G^{ε}_{k3}

Imperial College London

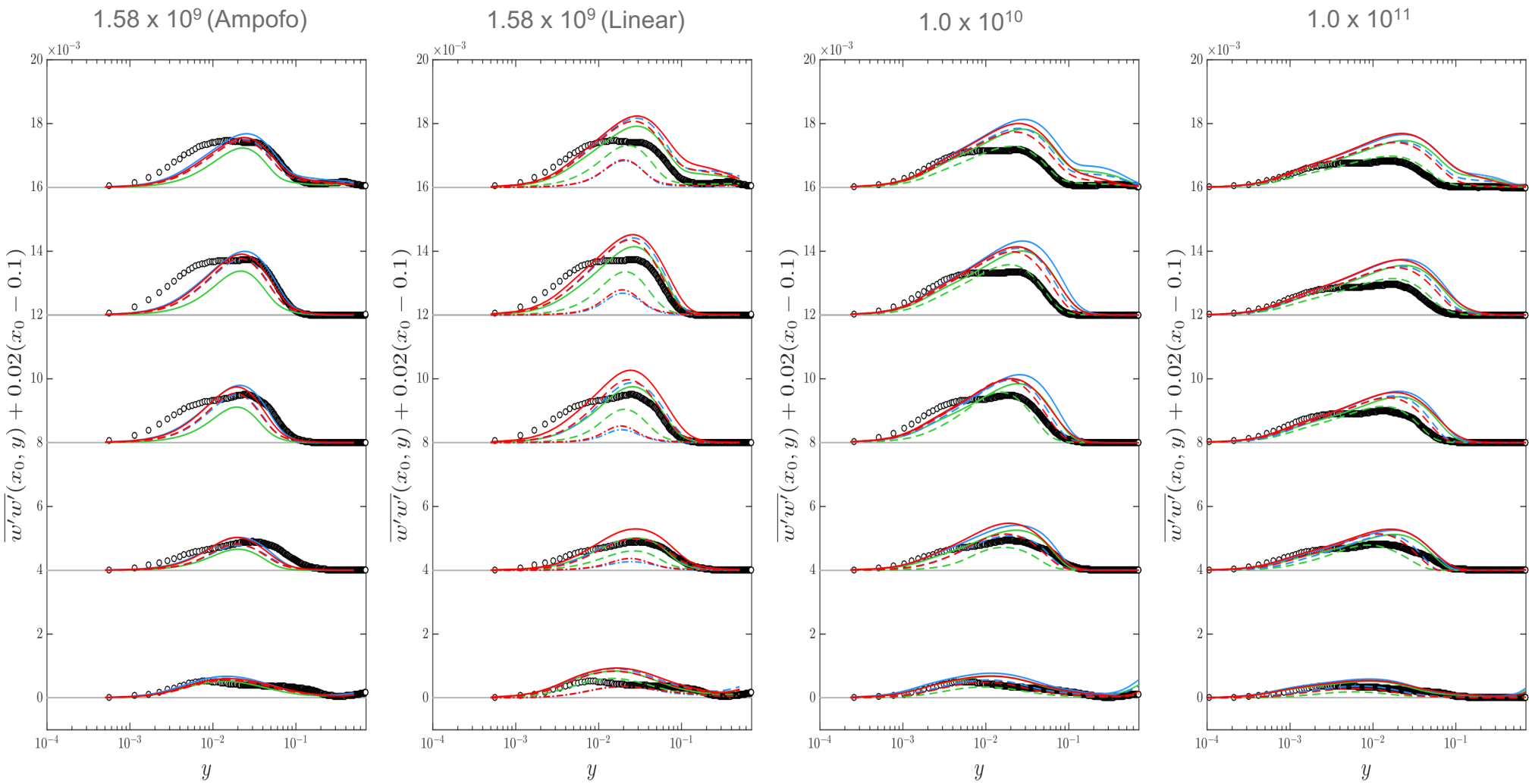
Comparison of Reynolds stresses: vv



○ DNS — AFM — EBAFM — EBDFM — G^{ϵ}_{k1} - - - G^{ϵ}_{k2} -·- G^{ϵ}_{k3}

Imperial College London

Comparison of Reynolds stresses: ww

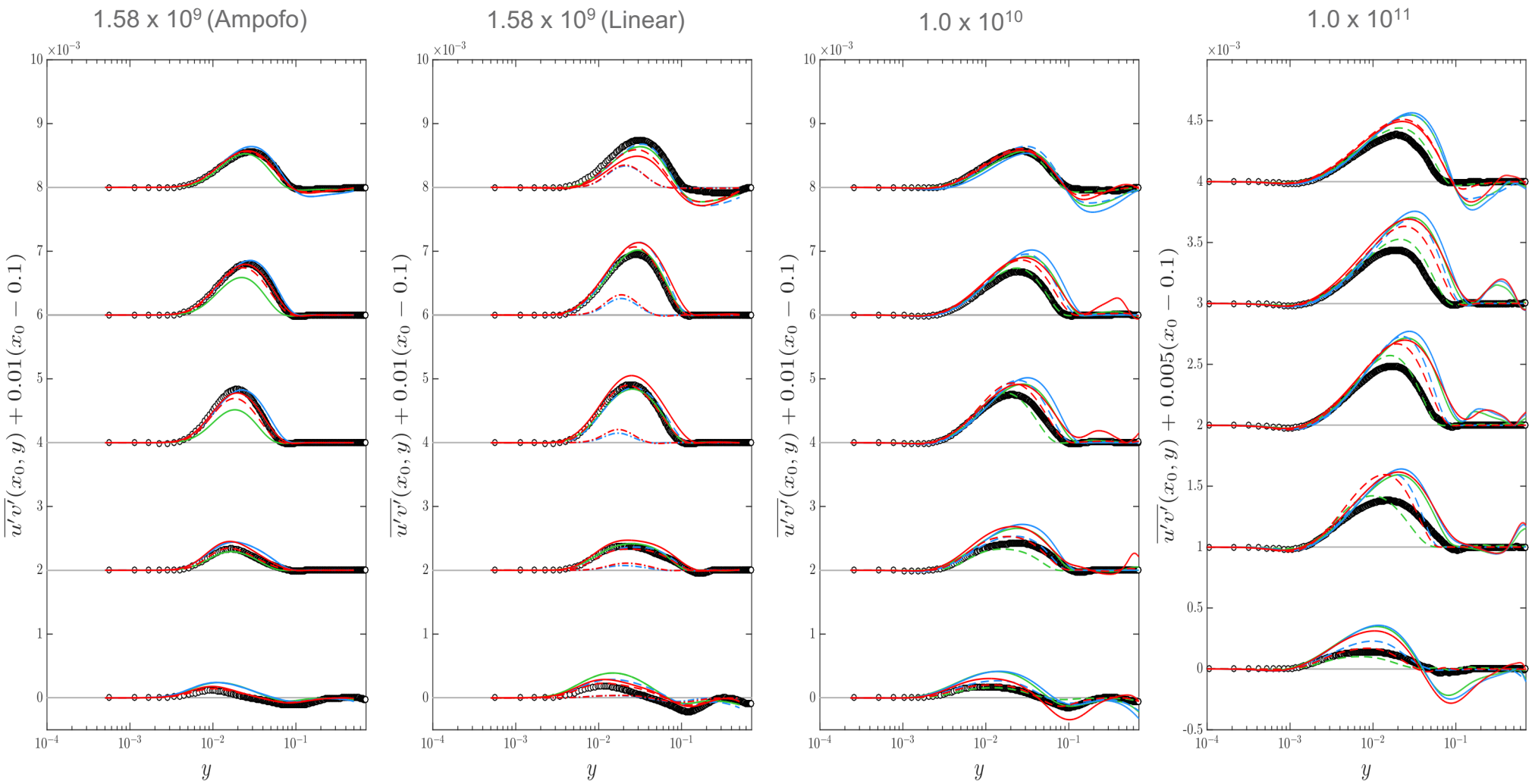


○ DNS — AFM — EBAFM — EBDFM

— G^{ε}_{k1} - - - G^{ε}_{k2} -·- G^{ε}_{k3}

Imperial College London

Comparison of Reynolds stresses: uv



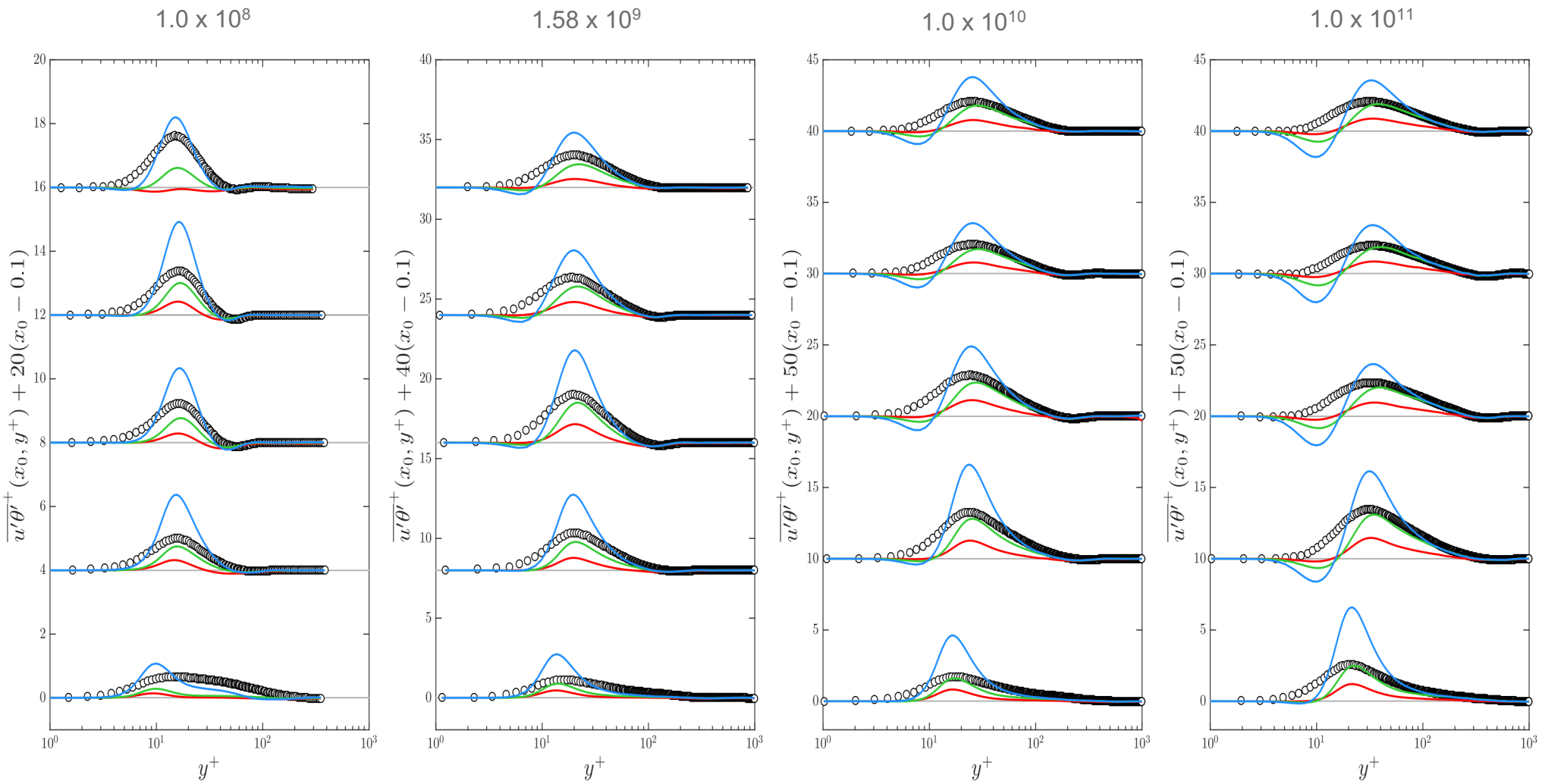
○ DNS — AFM — EBAFM — EBDFM

— G^{ϵ}_{k1} - - - G^{ϵ}_{k2} —·— G^{ϵ}_{k3}

A-priori analysis of EBRSM closure relations

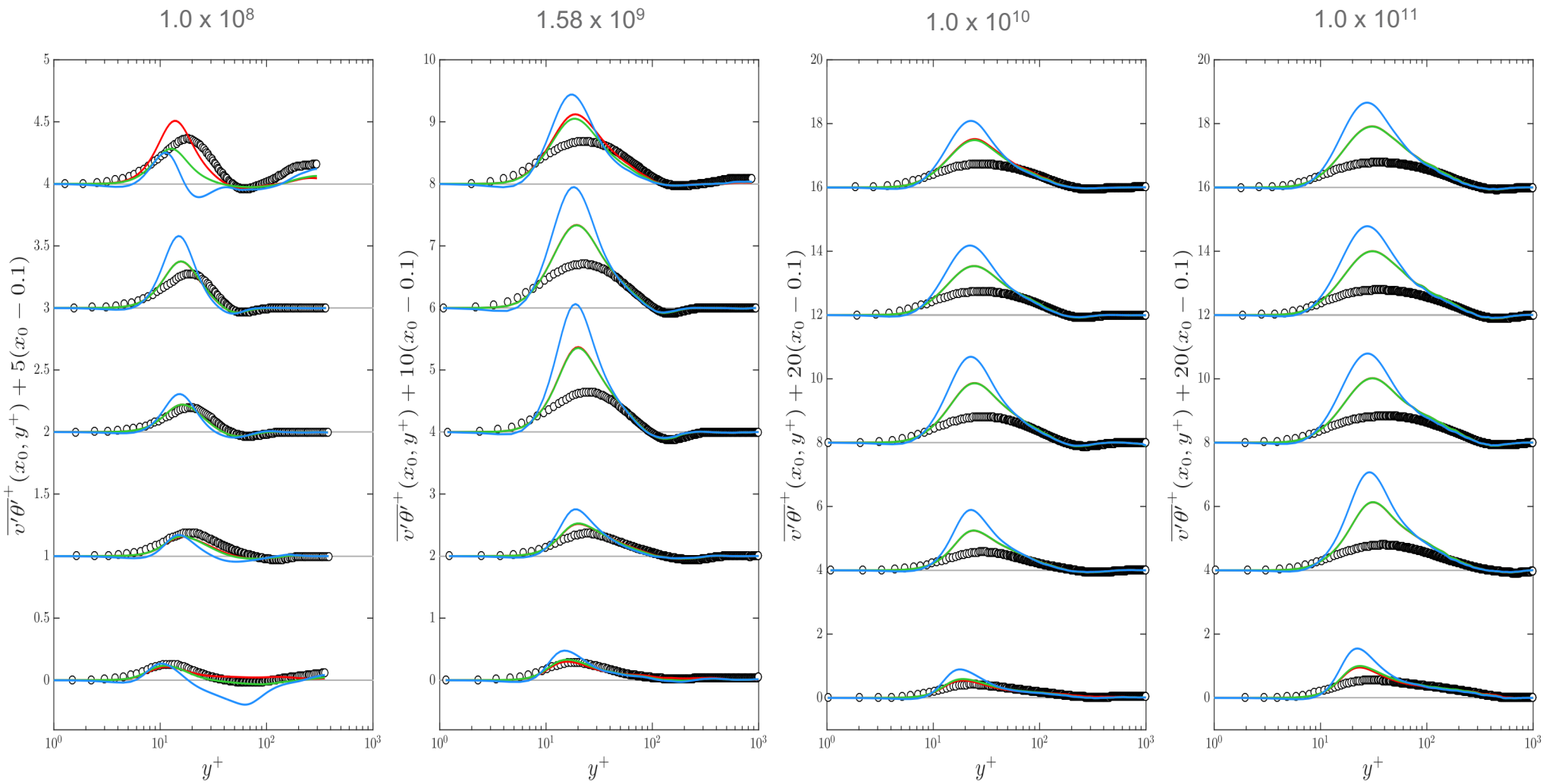
Imperial College London

Algebraic flux models: uT



○ *exact* $\overline{u'\theta'}$ — *estimated* GGDH — *estimated* AFM — *estimated* EBAFM

Algebraic flux models: vT

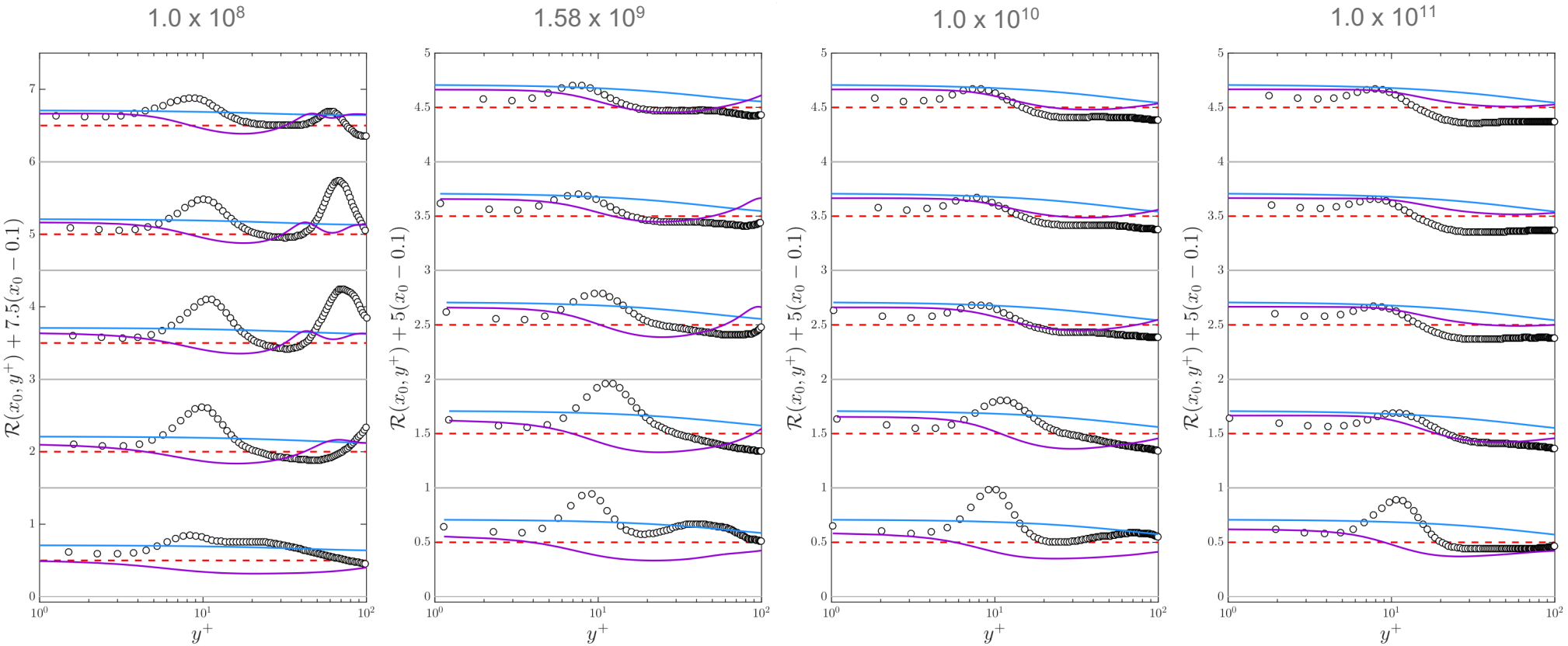


○ *exact* $\overline{v'\theta'}$ — *estimated* GGDH — *estimated* AFM — *estimated* EBAFM

Turbulent time scale ratio

Comparison of different formulations of the turbulent timescale ratio:

$$R_1 = R^h = 0.5 \qquad R_2 = (1-\alpha_\theta)\mathrm{Pr} + \alpha_\theta R^h \qquad R_3 = \frac{1}{1.5(1+A_{2\theta})}$$



○ DNS — — — $\mathcal{R}_1$ ——— $\mathcal{R}_2$ ——— $\mathcal{R}_3$